THE ALGEBRA

MOHAMMED BEN
PREFACE.

In the study of history, the attention of the observer is drawn by a peculiar charm towards those epochs, at which nations, after having secured their independence externally, strive to obtain an inward guarantee for their power, by acquiring eminence as great in science and in every art of peace as they have already attained in the field of war. Such an epoch was, in the history of the Arabs, that of the Caliphs Al Mansur, Harun al Rashid, and Al Mamun, the illustrious contemporaries of Charlemagne; to the glory of which era, in the volume now offered to the public, a new monument is endeavoured to be raised.

Abu Abdallah Mohammed ben Musa, of Khowarezm, who it appears, from his preface, wrote this Treatise at the command of the Caliph Al Mamun, was for a long time considered as the original inventor of Algebra. "Hæc ars olim a Mahomete, Mosis Arabis filio, initium sumsit: etenim hujus rei locuples testis Leo-
NARDUS PISANUS.” Such are the words with which Hieronymus Cardanus commences his Ars Magna, in which he frequently refers to the work here translated, in a manner to leave no doubt of its identity.

That he was not the inventor of the Art, is now well established; but that he was the first Mohammedan who wrote upon it, is to be found asserted in several Oriental writers. Haji Khalfa, in his bibliographical work, cites the initial words of the treatise now before us,* and

* I am indebted to the kindness of my friend Mr. Gustav Fluegel of Dresden, for a most interesting extract from this part of Haji Khalfa's work. Complete manuscript copies of the كشف الطنون are very scarce. The only two which I have hitherto had an opportunity of examining (the one bought in Egypt by Dr. Ehrenberg, and now deposited in the Royal Library at Berlin—the other among Rich's collection in the British Museum) are only abridgments of the original compilation, in which the quotation of the initial words of each work is generally omitted. The prospect of an edition and Latin translation of the complete original work, to be published by Mr. Fluegel, under the auspices of the Oriental Translation Committee, must under such circumstances be most gratifying to all friends of Asiatic literature.
states, in two distinct passages, that its author, **Mohammed ben Musa**, was the first Mussulman who had ever written on the solution of problems by the rules of completion and reduction. Two marginal notes in the Oxford manuscript—from which the text of the present edition is taken—and an anonymous Arabic writer, whose *Bibliotheca Philosophorum* is frequently quoted by **Casiri**,* likewise maintain that this production of **Mohammed ben Musa** was the first work written on the subject† by a Mohammedan.

* تاريُخ الحکماء, written in the twelfth century. **Casiri**

**Bibliotheca Arabica Escurialensis**, t. 1. 426. 428.

† The first of these marginal notes stands at the top of the first page of the manuscript, and reads thus: **هذا أول كتاب وضع في الجبر والمقابلة في الإسلام ولهذا ذكر فيه كل شيء طردنا لينفي الأصول في الجبر والمقابلة** "This is the first book written on (the art of calculating by) completion and reduction by a Mohammedan: on this account the author has introduced into it rules of various kinds, in order to render useful the very rudiments of Algebra." The other scholium stands farther on: it is the same to which I have referred in my notes to the Arabic text, p. 177.
From the manner in which our author, in his preface, speaks of the task he had undertaken, we cannot infer that he claimed to be the inventor. He says that the Caliph Al Mamun encouraged him to write a popular work on Algebra: an expression which would seem to imply that other treatises were then already extant. From a formula for finding the circumference of the circle, which occurs in the work itself (Text p. 51, Transl. p. 72), I have, in a note, drawn the conclusion, that part of the information comprised in this volume was derived from an Indian source; a conjecture which is supported by the direct assertion of the author of the Bibliotheca Philosophorum quoted by Casiri (1.426, 428). That Mohammed ben Musa was conversant with Hindu science, is further evident from the fact* that he abridged, at Al Mamun's request—but before the accession of that prince to the caliphat—the Sindhind, or

astronomical tables, translated by Mohammed ben Ibrahim al Fazari from the work of an Indian astronomer who visited the court of Almansur in the 156th year of the Hejira (A.D. 773).

The science as taught by Mohammed ben Musa, in the treatise now before us, does not extend beyond quadratic equations, including problems with an affected square. These he solves by the same rules which are followed by Diophantus*, and which are taught, though less comprehensively, by the Hindu mathematicians†. That he should have borrowed from Diophantus is not at all probable; for it does not appear that the Arabs had any knowledge of Diophantus' work before the middle of the fourth century after the Hejira, when Abu'l-Wafa Buzjani rendered it into Arabic‡. It

* See Diophantus, Introd. § ii. and Book iv. problems 32 and 33.
† Lilavati, p. 29, Vijaganita, p. 347, of Mr. Colebrooke's translation.
is far more probable that the Arabs received their first knowledge of Algebra from the Hindus, who furnished them with the decimal notation of numerals, and with various important points of mathematical and astronomical information.

But under whatever obligation our author may be to the Hindus, as to the subject matter of his performance, he seems to have been independent of them in the manner of digesting and treating it: at least the method which he follows in expounding his rules, as well as in showing their application, differs considerably from that of the Hindu mathematical writers. Bhaskara and Brahmagupta give dogmatical precepts, unsupported by argument, which, even by the metrical form in which they are expressed, seem to address themselves rather to the memory than to the reasoning faculty of the learner: Mohammed gives his rules in simple prose, and establishes their accuracy by geometrical illustrations. The Hindus give comparatively few examples, and are fond of investing the statement of their problems in
rhetorical pomp: the Arab, on the contrary, is remarkably rich in examples, but he introduces them with the same perspicuous simplicity of style which distinguishes his rules. In solving their problems, the Hindus are satisfied with pointing at the result, and at the principal intermediate steps which lead to it: the Arab shows the working of each example at full length, keeping his view constantly fixed upon the two sides of the equation, as upon the two scales of a balance, and showing how any alteration in one side is counterpoised by a corresponding change in the other.

Besides the few facts which have already been mentioned in the course of this preface, little or nothing is known of our Author's life. He lived and wrote under the caliphat of Al Mamun, and must therefore be distinguished from Abu Jafar Mohammed ben Musa*,

* The father of the latter, Musa ben Shaker, whose native country I do not find recorded, had been a robber or bandit in the earlier part of his life, but had afterwards found means to attach himself to the court of the Caliph Al-Mamun; who, after Musa’s death, took care of

the education of his three sons, Mohammed, Ahmed, and Al Hassan. (Abilfaragi Histor. Dyn. p. 280. Casiri, i. 386. 418). Each of the sons subsequently distinguished himself in mathematics and astronomy. We learn from Abulfaraj (l. c. p. 281) and from Ebn Khallikan (art. Thabet ben Korrah) that Thabet ben Korrah, the well-known translator of the Almagest, was indebted to Mohammed for his introduction to Al Motaded, and the men of science at the court of that caliph. Ebn Khallikan's words are:

"(Thabet ben Korrah) left Harran, and established himself at Kafratutha, where he remained till Mohammed ben Musa arrived there, on his return from the Greek dominions to Bagdad. The latter became acquainted with Thabet and on seeing his skill and sagacity, invited Thabet to accompany him to Bagdad, where Mohammed made him lodge at his own house, introduced him to the Caliph, and procured him an appointment in the body of astronomers." Ebn Khallikan here speaks of Mohammed ben Musa as of a well-known individual: he has however devoted no special article to an account of his life. It is possible
The manuscript from whence the text of the present edition is taken—and which is the only copy the existence of which I have as yet been able to trace—is preserved in the Bodleian collection at Oxford. It is, together with three other treatises on Arithmetic and Algebra, contained in the volume marked CMXVIII. Hunt. 214, fol., and bears the date of the transcription A.H. 743 (A.D. 1342). It is written in a plain and legible hand, but unfortunately destitute of most of the diacritical points: a deficiency which has often been very sensibly felt; for though the nature of the subject matter can but seldom leave a doubt as to the general import of a sentence, yet the true reading of some passages, and the precise interpretation of others, remain involved in obscurity. Besides, there occur several omissions of words, and even of entire sentences; and also instances of words or short passages writ-

that the tour into the provinces of the Eastern Roman Empire here mentioned, was undertaken in search of some ancient Greek works on mathematics or astronomy.
ten twice over, or words foreign to the sense introduced into the text. In printing the Arabic part, I have included in brackets many of those words which I found in the manuscript, the genuineness of which I suspected, and also such as I inserted from my own conjecture, to supply an apparent hiatus.

The margin of the manuscript is partially filled with scholia in a very small and almost illegible character, a few specimens of which will be found in the notes appended to my translation. Some of them are marked as being extracted from a commentary (شرح) by Al Mozaihafi*, probably the same author, whose full name is Jemaeddin Abu Abdallah Mohammed ben Omar al Jaza’i† al Mozaihafi, and whose “Introduction to Arithmetic,” (مقدمة في الحساب) is contained in the same volume with Mohammed’s work in the Bodleian library.

Numerals are in the text of the work always

---

* Wherever I have met with this name, it is written without the diacritical points المرجعى, and my pronunciation rests on mere conjecture.

† المرجعى (؟)
expressed by words: figures are only used in some of the diagrams, and in a few marginal notes.

The work had been only briefly mentioned in Uris' catalogue of the Bodleian manuscripts. Mr. H. T. Colebrooke first introduced it to more general notice, by inserting a full account of it, with an English translation of the directions for the solution of equations, simple and compound, into the notes of the "Dissertation" prefixed to his invaluable work, "Algebra, with Arithmetic and Mensuration, from the Sanscrit of Brahmegupta and Bhascara." (London, 1817, 4to. pages lxxv-lxxix.)

The account of the work given by Mr. Colebrooke excited the attention of a highly distinguished friend of mathematical science, who encouraged me to undertake an edition and translation of the whole: and who has taken the kindest interest in the execution of my task. He has with great patience and care revised and corrected my translation, and has furnished the commentary, subjoined to the text, in the form of common algebraic notation. But my
obligations to him are not confined to this only; for his luminous advice has enabled me to overcome many difficulties, which, to my own limited proficiency in mathematics, would have been almost insurmountable.

In some notes on the Arabic text which are appended to my translation, I have endeavoured, not so much to elucidate, as to point out for further enquiry, a few circumstances connected with the history of Algebra. The comparisons drawn between the Algebra of the Arabs and that of the early Italian writers might perhaps have been more numerous and more detailed; but my enquiry was here restricted by the want of some important works. Montucla, Cossali, Hutton, and the Basil edition of Cardanus' Ars magna, were the only sources which I had the opportunity of consulting.
THE AUTHOR'S PREFACE.

IN THE NAME OF GOD, GRACIOUS AND MERCIFUL!

This work was written by Mohammed ben Musa, of Khowarezm. He commences it thus:

Praised be God for his bounty towards those who deserve it by their virtuous acts: in performing which, as by him prescribed to his adoring creatures, we express our thanks, and render ourselves worthy of the continuance (of his mercy), and preserve ourselves from change: acknowledging his might, bending before his power, and revering his greatness! He sent Mohammed (on whom may the blessing of God repose!) with the mission of a prophet, long after any messenger from above had appeared, when justice had fallen into neglect, and when the true way of life was sought for in vain. Through him he cured of blindness, and saved through him from perdition, and increased
through him what before was small, and collected through him what before was scattered. Praised be God our Lord! and may his glory increase, and may all his names be hallowed—besides whom there is no God; and may his benediction rest on Mohammed the Prophet and on his descendants!

The learned in times which have passed away, and among nations which have ceased to exist, were constantly employed in writing books on the several departments of science and on the various branches of knowledge, bearing in mind those that were to come after them, and hoping for a reward proportionate to their ability, and trusting that their endeavours would meet with acknowledgment, attention, and remembrance—content as they were even with a small degree of praise; small, if compared with the pains which they had undergone, and the difficulties which they had encountered in revealing the secrets and obscurities of science.

(2) Some applied themselves to obtain information which was not known before them, and left it to posterity; others commented upon the difficulties in the works left by their predecessors, and defined the best method (of study), or rendered the access (to science) easier or
placed it more within reach; others again discovered mistakes in preceding works, and arranged that which was confused, or adjusted what was irregular, and corrected the faults of their fellow-labourers, without arrogance towards them, or taking pride in what they did themselves.

That fondness for science, by which God has distinguished the Imam al Mamun, the Commander of the Faithful (besides the caliphat which He has vouchsafed unto him by lawful succession, in the robe of which He has invested him, and with the honours of which He has adorned him), that affability and condescension which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, —has encouraged me to compose a short work on Calculating by (the rules of) Completion and Reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned—relying on the good-
ness of my intention therein, and hoping that the learned will reward it, by obtaining (for me) through their prayers the excellence of the Divine mercy: in requital of which, may the choicest blessings and the abundant bounty of God be theirs! My confidence rests with God, in this as in every thing, and in Him I put my trust. He is the Lord of the Sublime Throne. May His blessing descend upon all the prophets and heavenly messengers!
When I considered what people generally want in calculating, I found that it always is a number. I also observed that every number is composed of units, and that any number may be divided into units. Moreover, I found that every number, which may be expressed from one to ten, surpasses the preceding by one unit: afterwards the ten is doubled or tripled, just as before the units were: thus arise twenty, thirty, &c., until a hundred; then the hundred is doubled and tripled in the same manner as the units and the tens, up to a thousand; then the thousand can be thus repeated at any complex number; and so forth to the utmost limit of numeration.

I observed that the numbers which are required in calculating by Completion and Reduction are of three kinds, namely, roots, squares, and simple numbers relative to neither root nor square.
A root is any quantity which is to be multiplied by itself, consisting of units, or numbers ascending, or fractions descending. *

A square is the whole amount of the root multiplied by itself.

A simple number is any number which may be pronounced without reference to root or square.

A number belonging to one of these three classes may be equal to a number of another class; you may say, for instance, "squares are equal to roots," or "squares are equal to numbers," or "roots are equal to numbers."†

Of the case in which squares are equal to roots, this is an example. "A square is equal to five roots of the same;"‡ the root of the square is five, and the square is twenty-five, which is equal to five times its root.

So you say, "one third of the square is equal to four roots;"§ then the whole square is equal to twelve roots; that is a hundred and forty-four; and its root is twelve.

Or you say, "five squares are equal to ten roots;"‖ then one square is equal to two roots; the root of the square is two, and its square is four.

* By the word root, is meant the simple power of the unknown quantity.

† \[ cx^2 = bx \]
\[ cx^2 = a \]
\[ bx = a \]

‡ \[ x^2 = 5x \]
\[ \therefore x = 5 \]

§ \[ \frac{x^2}{3} = 4x \]
\[ \therefore \frac{x^2}{3} = 12x \]
\[ \therefore x = 12 \]

‖ \[ 5x^2 = 10x \]
\[ \therefore x^2 = 2x \]
\[ \therefore x = 2 \]
In this manner, whether the squares be many or few, 
(i. e. multiplied or divided by any number), they are 
reduced to a single square; and the same is done with 
the roots, which are their equivalents; that is to say, 
they are reduced in the same proportion as the squares.
As to the case in which squares are equal to numbers; 
for instance, you say, "a square is equal to nine;"* 
then this is a square, and its root is three. Or "five 
squares are equal to eighty;"† then one square is equal 
to one-fifth of eighty, which is sixteen. Or "the half 
of the square is equal to eighteen;"‡ then the square is 
thirty-six, and its root is six.

Thus, all squares, multiples, and sub-multiples of 
them, are reduced to a single square. If there be only 
part of a square, you add thereto, until there is a whole 
square; you do the same with the equivalent in numbers.

As to the case in which roots are equal to numbers; 
for instance, "one root equals three in number;"§ then 
the root is three, and its square nine. Or "four roots (5) 
are equal to twenty;"‖ then one root is equal to five, 
and the square to be formed of it is twenty-five. 
Or "half the root is equal to ten;"‖‖ then the

* \( x^2 = 9 \) \( \therefore x = 3 \)
† \( 5x^2 = 80 \therefore x^2 = \frac{80}{5} = 16 \)
‡ \( \frac{x^2}{v} = 18 \therefore x^2 = 36 \therefore x = 6 \)
§ \( x = 3 \)
‖ \( 4x = 20 \therefore x = 5 \)
‖‖ \( \frac{x}{2} = 10 \therefore x = 20 \)
whole root is equal to twenty, and the square which is formed of it is four hundred.

I found that these three kinds; namely, roots, squares, and numbers, may be combined together, and thus three compound species arise;* that is, "squares and roots equal to numbers;" "squares and numbers equal to roots;" "roots and numbers equal to squares."

Roots and Squares are equal to Numbers;† for instance, "one square, and ten roots of the same, amount to thirty-nine dirhems;" that is to say, what must be the square which, when increased by ten of its own roots, amounts to thirty-nine? The solution is this: you halve the number‡ of the roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this, which is eight, and subtract from it half the number of the roots, which is five; the remainder is three. This is the root of the square which you sought for; the square itself is nine.

* The three cases considered are,

1st. \( cx^2 + bx = a \)
2d. \( cx^2 = a = bx \)
3d. \( cx^2 = bx + a \)

† 1st case: \( cx^2 + bx = a \)
Example \( x^2 + 10x = 39 \)
\( x = \sqrt{\left( \frac{1}{2} \right)^2 + 39} - \frac{10}{2} \)
\( = \sqrt{64} - 5 \)
\( = 8 - 5 = 3 \)
‡ i.e. the coefficient.
The solution is the same when two squares or three, or more or less be specified;* you reduce them to one single square, and in the same proportion you reduce also the roots and simple numbers which are connected therewith.

For instance, "two squares and ten roots are equal to forty-eight dirhems;"† that is to say, what must be the amount of two squares which, when summed up and added to ten times the root of one of them, make up a sum of forty-eight dirhems? You must at first reduce the two squares to one; and you know that one square of the two is the moiety of both. Then reduce everything mentioned in the statement to its half, and it will be the same as if the question had been, a square and five roots of the same are equal to twenty-four dirhems; or, what must be the amount of a square which, when added to five times its root, is equal to twenty-four dirhems? Now halve the number of the roots; the moiety is two and a half. Multiply that by itself; the product is six and a quarter. Add this to twenty-four; the sum is thirty dirhems and a quarter. Take the root of this; it is five and a half. Subtract from this the moiety of the number of the roots, that is two and a half; the

\[ cx^2 + bx = a \]

is to be reduced to the form \[ x^2 + \frac{b}{c} x = \frac{a}{c} \]

\[ 2x^2 + 10x = 48 \]

\[ x^2 + 5x = 24 \]

\[ x = \sqrt{\left(\frac{5}{2}\right)^2 + 24} - \frac{5}{2} \]

\[ = \sqrt{\left[\frac{64}{4} + 24\right]} - 2\frac{1}{2} \]

\[ = 5\frac{1}{2} - 2\frac{1}{2} = 3 \]
remainder is three. This is the root of the square, and the square itself is nine.

The proceeding will be the same if the instance be, "half of a square and five roots are equal to twenty-eight dirhems;"* that is to say, what must be the amount of a square, the moiety of which, when added to the equivalent of five of its roots, is equal to twenty-eight dirhems? Your first business must be to complete your square, so that it amounts to one whole square. This you effect by doubling it. Therefore double it, and double also that which is added to it, as well as what is equal to it. Then you have a square and ten roots, equal to fifty-six dirhems. Now halve the roots; the moiety is five. Multiply this by itself; the product is twenty-five. Add this to fifty-six; the sum is eighty-one. Extract the root of this; it is nine. Subtract from this the moiety of the number of roots, which is five; the remainder is four. This is the root of the square which you sought for; the square is sixteen, and half the square eight.

Proceed in this manner, whenever you meet with squares and roots that are equal to simple numbers: for it will always answer.

\[
\begin{align*}
x^2 + 5x &= 28 \\
x^2 + 10x &= 56 \\
&= \sqrt{\left(\frac{10}{2}\right)^2 + 56} - \frac{10}{2} \\
&= \sqrt{25 + 56} - 5 \\
&= \sqrt{81} - 5 \\
&= 9 - 5 = 4
\end{align*}
\]
Squares and Numbers are equal to Roots;* for instance, “a square and twenty-one in numbers are equal to ten roots of the same square.” That is to say, what must be the amount of a square, which, when twenty-one dirhems are added to it, becomes equal to the equivalent of ten roots of that square? Solution: Halve the number of the roots; the moiety is five. Multiply this by itself; the product is twenty-five. Subtract from this the twenty-one which are connected with the square; the remainder is four. Extract its root; it is two. Subtract this from the moiety of the roots, which is five; the remainder is three. This is the root of the square which you required, and the square is nine. Or you may add the root to the moiety of the roots; the sum is seven; this is the root of the square which you sought for, and the square itself is forty-nine.

When you meet with an instance which refers you to this case, try its solution by addition, and if that do not serve, then subtraction certainly will. For in this case both addition and subtraction may be employed, which will not answer in any other of the three cases in which

* 2d case. \( cx^2 + a = bx \)
Example. \( x^2 + 21 = 10x \)
\[
x = \frac{10}{2} \pm \sqrt{\left(\frac{10}{2}\right)^2 - 21}
\]
\[
= 5 \pm \sqrt{25 - 21}
\]
\[
= 5 \pm \sqrt{4}
\]
\[
= 5 \pm 2
\]
the number of the roots must be halved. And know, that, when in a question belonging to this case you have halved the number of the roots and multiplied the moiety by itself, if the product be less than the number of dirhems connected with the square, then the instance is impossible;* but if the product be equal to the dirhems by themselves, then the root of the square is equal to the moiety of the roots alone, without either addition or subtraction.

In every instance where you have two squares, or more or less, reduce them to one entire square, † as I have explained under the first case.

* If in an equation, of the form \( x^2 + a = bx \), \((\frac{b}{2})^2 < a\), the case supposed in the equation cannot happen. If \((\frac{b}{2})^2 = a\), then \( x = \frac{b}{2} \)

† \( cx^2 + a = bx \) is to be reduced to \( x^2 + \frac{a}{c} = \frac{b}{c}x \)

‡ 3d case \( cx^2 = bx + a \)

Example \( x^2 = 3x + 4 \)

\[
x^2 = \sqrt{[(\frac{3}{2})^2 + 4]} + \frac{3}{2}
= \sqrt{1 \frac{1}{4}} + 4 + 1 \frac{1}{2}
= \sqrt{2 \frac{1}{4}} + 4 + 1 \frac{1}{2}
= \sqrt{6 \frac{1}{4}} + 1 \frac{1}{2}
= 2 \frac{1}{2} + 1 \frac{1}{2} = 4
\]
six and a quarter. Extract its root; it is two and a half. Add this to the moiety of the roots, which was one and a half; the sum is four. This is the root of the square, and the square is sixteen.

Whenever you meet with a multiple or sub-multiple of a square, reduce it to one entire square.

These are the six cases which I mentioned in the introduction to this book. They have now been explained. I have shown that three among them do not require that the roots be halved, and I have taught how they must be resolved. As for the other three, in which halving the roots is necessary, I think it expedient, more accurately, to explain them by separate chapters, in which a figure will be given for each case, to point out the reasons for halving.

Demonstration of the Case: "a Square and ten Roots are equal to thirty-nine Dirhems."*

The figure to explain this a quadrate, the sides of which are unknown. It represents the square, the which, or the root of which, you wish to know. This is the figure A B, each side of which may be considered as one of its roots; and if you multiply one of these sides by any number, then the amount of that number may be looked upon as the number of the roots which are added to the square. Each side of the quadrate represents the root of the square; and, as in the instance,

* Geometrical illustration of the case, \( x^2 + 10x = 39 \)
the roots were connected with the square, we may take one-fourth of ten, that is to say, two and a half, and combine it with each of the four sides of the figure. Thus with the original quadrate A B, four new parallelograms are combined, each having a side of the quadrate as its length, and the number of two and a half as its breadth; they are the parallelograms C, G, T, and K. We have now a quadrate of equal, though unknown sides; but in each of the four corners of which a square piece of two and a half multiplied by two and a half is wanting. In order to compensate for this want and to complete the quadrate, we must add (to that which we have already) four times the square of two and a half, that is, twenty-five. We know (by the statement) that the first figure, namely, the quadrate representing the square, together with the four parallelograms around it, which represent the ten roots, is equal to thirty-nine of numbers. If to this we add twenty-five, which is the equivalent of the four quadrates at the corners of the figure A B, by which the great figure D H is completed, then we know that this together makes sixty-four. One side of this great quadrate is its root, that is, eight. If we subtract twice a fourth of ten, that is five, from eight, as from the two extremities of the side of the great quadrate D H, then the remainder of such a side will be three, and that is the root of the square, or the side of the original figure A B. It must be observed, that we have halved the number of the roots, and added the product of the moiety multiplied by itself to the number
thirty-nine, in order to complete the great figure in its four corners; because the fourth of any number multiplied by itself, and then by four, is equal to the product of the moiety of that number multiplied by itself.* Accordingly, we multiplied only the moiety of the roots by itself, instead of multiplying its fourth by itself, and then by four. This is the figure:

![Diagram](https://example.com/diagram.png)

The same may also be explained by another figure. We proceed from the quadrate A B, which represents the square. It is our next business to add to it the ten roots of the same. We halve for this purpose the ten, so that it becomes five, and construct two quadrangles on two sides of the quadrate A B, namely, G and D, the length of each of them being five, as the moiety of the ten roots, whilst the breadth of each is equal to a side of the quadrate A B. Then a quadrate remains opposite the corner of the quadrate A B. This is equal to five multiplied by five: this five being half of the number of the roots which we have added to each of the two sides of the first quadrate. Thus we know that

\[ 4 \times \left(\frac{b}{4}\right)^2 = \left(\frac{b}{2}\right)^2 \]
the first quadrate, which is the square, and the two quadrangles on its sides, which are the ten roots, make together thirty-nine. In order to complete the great quadrate, there wants only a square of five multiplied by five, or twenty-five. This we add to thirty-nine, in order to complete the great square \( S H \). The sum is sixty-four. We extract its root, eight, which is one of the sides of the great quadrangle. By subtracting from this the same quantity which we have before added, namely five, we obtain three as the remainder. This is the side of the quadrangle \( A B \), which represents the square; it is the root of this square, and the square itself is nine. This is the figure:

![Diagram](image)

*Demonstration of the Case: “a Square and twenty-one Dirhems are equal to ten Roots.”*

We represent the square by a quadrate \( AD \), the length of whose side we do not know. To this we join a parallelogram, the breadth of which is equal to one of the sides of the quadrate \( AD \), such as the side \( HN \). This parallelogram is \( HB \). The length of the two

\[ x^2 + 21 = 10x \]

* Geometrical illustration of the case,
figures together is equal to the line H C. We know that its length is ten of numbers; for every quadrate has equal sides and angles, and one of its sides multiplied by a unit is the root of the quadrate, or multiplied by two it is twice the root of the same. As it is stated, therefore, that a square and twenty-one of numbers are equal to ten roots, we may conclude that the length of the line H C is equal to ten of numbers, since the line C D represents the root of the square. We now divide the line C H into two equal parts at the point G: the line G C is then equal to H G. It is also evident that (12) the line G T is equal to the line C D. At present we add to the line G T, in the same direction, a piece equal to the difference between C G and G T, in order to complete the square. Then the line T K becomes equal to K M, and we have a new quadrate of equal sides and angles, namely, the quadrate M T. We know that the line T K is five; this is consequently the length also of the other sides: the quadrate itself is twenty-five, this being the product of the multiplication of half the number of the roots by themselves, for five times five is twenty-five. We have perceived that the quadrangle H B represents the twenty-one of numbers which were added to the quadrate. We have then cut off a piece from the quadrangle H B by the line K T (which is one of the sides of the quadrate M T), so that only the part T A remains. At present we take from the line K M the piece K L, which is equal to G K; it then appears that the line T G is equal to M L; more-
over, the line KL, which has been cut off from KM, is equal to KG; consequently, the quadrangle MR is equal to TA. Thus it is evident that the quadrangle HT, augmented by the quadrangle MR, is equal to the quadrangle HB, which represents the twenty-one. The whole quadrate MT was found to be equal to twenty-five. If we now subtract from this quadrate, MT, the quadrangles HT and MR, which are equal to twenty-one, there remains a small quadrate KR, which represents the difference between twenty-five and twenty-one. This is four; and its root, represented by the line RG, which is equal to GA, is two. If you (13) subtract this number two from the line CG, which is the moiety of the roots, then the remainder is the line AC; that is to say, three, which is the root of the original square. But if you add the number two to the line CG, which is the moiety of the number of the roots, then the sum is seven, represented by the line CR, which is the root to a larger square. However, if you add twenty-one to this square, then the sum will likewise be equal to ten roots of the same square. Here is the figure:

![Diagram](image-url)
Demonstration of the Case: "three Roots and four of Simple Numbers are equal to a Square."

Let the square be represented by a quadrangle, the sides of which are unknown to us, though they are equal among themselves, as also the angles. This is the quadrato A D, which comprises the three roots and the four of numbers mentioned in this instance. In every quadrato one of its sides, multiplied by a unit, is its root. We now cut off the quadrangle H D from the quadrato A D, and take one of its sides H C for three, which is the number of the roots. The same is equal to R D. It follows, then, that the quadrangle H B represents the four of numbers which are added to the roots. Now we halve the side C H, which is equal to three roots, at the point G; from this division we construct the square H T, which is the product of half the roots (or one and (14) a half) multiplied by themselves, that is to say, two and a quarter. We add then to the line G T a piece equal to the line A H, namely, the piece T L; accordingly the line G L becomes equal to A G, and the line K N equal to T L. Thus a new quadrangle, with equal sides and angles, arises, namely, the quadrangle G M; and we find that the line A G is equal to M L, and the same line A G is equal to G L. By these means the line C G remains equal to N R, and the line M N equal to T L, and from the quadrangle H B a piece equal to the quadrangle K L is cut off.

* Geometrical illustration of the 3d case, \( x^2 = 3x + 4 \)
But we know that the quadrangle $AR$ represents the four of numbers which are added to the three roots. The quadrangle $AN$ and the quadrangle $KL$ are together equal to the quadrangle $AR$, which represents the four of numbers.

We have seen, also, that the quadrangle $GM$ comprises the product of the moiety of the roots, or of one and a half, multiplied by itself; that is to say two and a quarter, together with the four of numbers, which are represented by the quadrangles $AN$ and $KL$. There remains now from the side of the great original quadrate $AD$, which represents the whole square, only the moiety of the roots, that is to say, one and a half, namely, the line $GC$. If we add this to the line $AG$, which is the root of the quadrate $GM$, being equal to two and a half; then this, together with $CG$, or the moiety of the three roots, namely, one and a half, makes four, which is the line $AC$, or the root to a square, which is represented by the quadrate $AD$. Here follows the figure. This it was which we were desirous to explain.
We have observed that every question which requires equation or reduction for its solution, will refer you to one of the six cases which I have proposed in this book. I have now also explained their arguments. Bear them, therefore, in mind.

---

ON MULTIPLICATION.

I shall now teach you how to multiply the unknown numbers, that is to say, the roots, one by the other, if they stand alone, or if numbers are added to them, or if numbers are subtracted from them, or if they are subtracted from numbers; also how to add them one to the other, or how to subtract one from the other.

Whenever one number is to be multiplied by another, the one must be repeated as many times as the other contains units.*

If there are greater numbers combined with units to be added to or subtracted from them, then four multiplications are necessary;† namely, the greater numbers by the greater numbers, the greater numbers by the

* If \( x \) is to be multiplied by \( y \), \( x \) is to be repeated as many times as there are units in \( y \).

† If \( x \pm a \) is to be multiplied by \( y \pm b \), \( x \) is to be multiplied by \( y \), \( x \) is to be multiplied by \( b \), \( a \) is to be multiplied by \( y \), and \( a \) is to be multiplied by \( b \).
units, the units by the greater numbers, and the units by the units.

If the units, combined with the greater numbers, are positive, then the last multiplication is positive; if they are both negative, then the fourth multiplication is likewise positive. But if one of them is positive, and one (16) negative, then the fourth multiplication is negative.*

For instance, "ten and one to be multiplied by ten and two."† Ten times ten is a hundred; once ten is ten positive; twice ten is twenty positive, and once two is two positive; this altogether makes a hundred and thirty-two.

But if the instance is "ten less one, to be multiplied by ten less one,"‡ then ten times ten is a hundred; the

* In multiplying \((x \pm a)\) by \((y \pm b)\)

\[
\begin{align*}
+ax + b &= +ab \\
-ax - b &= +ab \\
+ax - b &= -ab \\
-ax + b &= -ab \\
\end{align*}
\]

† \((10 + 1)(10 + 2)\)

\[
\begin{align*}
= & 10 \times 10 \ldots 100 \\
+ & 1 \times 10 \ldots 10 \\
+ & 2 \times 10 \ldots 20 \\
+ & 1 \times 2 \ldots 2 \\
+ & 132 \\
\end{align*}
\]

‡ \((10 - 1)(10 - 1)\)

\[
\begin{align*}
= & 10 \times 10 \ldots + 100 \\
- & 1 \times 10 \ldots - 10 \\
- & 1 \times 10 \ldots - 10 \\
- & 1 \times -1 \ldots + 1 \\
+ & 81 \\
\end{align*}
\]
negative one by ten is ten negative; the other negative one by ten is likewise ten negative, so that it becomes eighty: but the negative one by the negative one is one positive, and this makes the result eighty-one.

Or if the instance be “ten and two, to be multiplied by ten less one,”* then ten times ten is a hundred, and the negative one by ten is ten negative; the positive two by ten is twenty positive; this together is a hundred and ten; the positive two by the negative one gives two negative. This makes the product a hundred and eight.

I have explained this, that it might serve as an introduction to the multiplication of unknown sums, when numbers are added to them, or when numbers are subtracted from them, or when they are subtracted from numbers.

For instance: “Ten less thing (the signification of thing being root) to be multiplied by ten.”† You begin by taking ten times ten, which is a hundred; less thing by ten is ten roots negative; the product is therefore a hundred less ten things.

\[
\begin{align*}
* \quad (10 + 2) \times (10 - 1) &= \\
&= 10 \times 10 \ldots \ldots 100 \\
&- 1 \times 10 \ldots \ldots -10 \\
&+ 10 \times 2 \ldots \ldots +20 \\
&- 1 \times 2 \ldots \ldots -2 \\
&= 108
\end{align*}
\]

† \( (10 - x) \times 10 = 10 \times 10 - 10x = 100 - 10x. \)
If the instance be: "ten and thing to be multiplied by ten,"* then you take ten times ten, which is a hundred, and thing by ten is ten things positive; so that the product is a hundred plus ten things.

If the instance be: "ten and thing to be multiplied (17) by itself,"† then ten times ten is a hundred, and ten times thing is ten things; and again, ten times thing is ten things; and thing multiplied by thing is a square positive, so that the whole product is a hundred dirhems and twenty things and one positive square.

If the instance be: "ten minus thing to be multiplied by ten minus thing,"‡ then ten times ten is a hundred; and minus thing by ten is minus ten things; and again, minus thing by ten is minus ten things. But minus thing multiplied by minus thing is a positive square. The product is therefore a hundred and a square, minus twenty things.

In like manner if the following question be proposed to you: "one dirhem minus one-sixth to be multiplied by one dirhem minus one-sixth;"§ that is to say, five-sixths by themselves, the product is five and twenty parts of a dirhem, which is divided into six and thirty parts, or two-thirds and one-sixth of a sixth. Computation: You multiply one dirhem by one dirhem, the

\[ *(10 + x) \times 10 = 10 \times 10 + 10x = 100 + 10x \]
\[ †(10 + x)(10 + x) = 10 \times 10 + 10x + 10x + x^2 = 100 + 20x + x^2 \]
\[ ‡(10 - x)(10 - x) = 10 \times 10 - 10x - 10x + x^2 = 100 - 20x + x^2 \]
\[ §(1 - \frac{1}{6})(1 - \frac{1}{6}) = 1 - \frac{1}{6} \times \frac{1}{6} = \frac{3}{6} + \frac{1}{6} \times \frac{1}{6}; \text{i.e.} \frac{5}{6} = \frac{3}{6} + \frac{1}{6} \times \frac{1}{6} \]
product is one dirhem; then one dirhem by minus one-sixth, that is one-sixth negative; then, again, one dirhem by minus one-sixth is one-sixth negative: so far, then, the result is two-thirds of a dirhem: but there is still minus one-sixth to be multiplied by minus one-sixth, which is one-sixth of a sixth positive; the product is, therefore, two-thirds and one sixth of a sixth.

If the instance be, "ten minus thing to be multiplied by ten and thing," then you say,* ten times ten is a hundred; and minus thing by ten is ten things negative; and thing by ten is ten things positive; and minus thing by thing is a square things positive; therefore, the product is a hundred dirhems, minus a square.

If the instance be, "ten minus thing to be multiplied by thing,"† then you say, ten multiplied by thing is ten things; and minus thing by thing is a square thing negative; (18) therefore, the product is ten things minus a square.

If the instance be, "ten and thing to be multiplied by thing less ten,"‡ then you say, thing multiplied by ten is ten things positive; and thing by thing is a square thing positive; and minus ten by ten is a hundred dirhems negative; and minus ten by thing is ten things negative. You say, therefore, a square minus a hundred dirhems; for, having made the reduction, that is to say, having removed the ten things positive by the ten things

\[ (10-x)(10+x)=10 \times 10-10x+10x-x^2=100-x^2 \]
\[ (10-x)\times x=10x-x^2 \]
\[ (10+x)(x-10)=10x+x^2-100-10x=x^2-100 \]
negative, there remains a square minus a hundred dirhems.

If the instance be, "ten dirhems and half a thing to be multiplied by half a dirhem, minus five things,"* then you say, half a dirhem by ten is five dirhems positive; and half a dirhem by half a thing is a quarter of thing positive; and minus five things by ten dirhems is fifty roots negative. This altogether makes five dirhems minus forty-nine things and three quarters of thing. After this you multiply five roots negative by half a root positive: it is two squares and a half negative. Therefore, the product is five dirhems, minus two squares and a half, minus forty-nine roots and three quarters of a root.

If the instance be, "ten and thing to be multiplied by thing less ten,"† then this is the same as if it were said thing and ten by thing less ten. You say, therefore, thing multiplied by thing is a square positive; and ten by thing is ten things positive; and minus ten by thing is ten things negative. You now remove the positive by the negative, then there only remains a square. Minus ten multiplied by ten is a hundred, to be subtracted from the square. This, therefore, altogether, is a square less a hundred dirhems.

(19) Whenever a positive and a negative factor concur in

\[
*(10 + \frac{x}{2})(\frac{1}{2} - 5x) = \frac{10}{2} + x - 50x - \frac{5}{2}x^2 = 5 - 49\frac{3}{4}x - 2\frac{1}{2}x^2
\]

†(10 + x)(x - 10) = (x + 10)(x - 10) = x^2 + 10x - 10x - 100 = x^2 - 100
a multiplication, such as thing positive and minus thing, the last multiplication gives always the negative product. Keep this in memory.

**ON ADDITION AND SUBTRACTION.**

Know that the root of two hundred minus ten, added to twenty minus the root of two hundred, is just ten.*

The root of two hundred, minus ten, subtracted from twenty minus the root of two hundred, is thirty minus twice the root of two hundred; twice the root of two hundred is equal to the root of eight hundred.†

A hundred and a square minus twenty roots, added to fifty and ten roots minus two squares,‡ is a hundred and fifty, minus a square and minus ten roots.

A hundred and a square, minus twenty roots, diminished by fifty and ten roots minus two squares, is fifty dirhems and three squares minus thirty roots.§

I shall hereafter explain to you the reason of this by a figure, which will be annexed to this chapter.

If you require to double the root of any known or unknown square, (the meaning of its duplication being

* \[20 - \sqrt{200} + (\sqrt{200} - 10) = 10\]
† \[20 - \sqrt{200} - (\sqrt{200} - 10) = 30 - 2\sqrt{200} = 30 - \sqrt{800}\]
‡ \[50 + 10x - 2x^2 + (100 + x^2 - 20x) = 150 - 10x - x^2\]
§ \[100 + x^2 - 20x - [50 - 2x^2 + 10x] = 50 + 3x^2 - 30x\]
that you multiply it by two) then it will suffice to multiply two by two, and then by the square;* the root of the product is equal to twice the root of the original square.

If you require to take it thrice, you multiply three by three, and then by the square; the root of the product is thrice the root of the original square.

Compute in this manner every multiplication of the roots, whether the multiplication be more or less than two.†

(20) If you require to find the moiety of the root of the square, you need only multiply a half by a half, which is a quarter; and then this by the square: the root of the product will be half the root of the first square.‡

Follow the same rule when you seek for a third, or a quarter of a root, or any larger or smaller quota§ of it, whatever may be the denominator or the numerator.

Examples of this: If you require to double the root of nine,|| you multiply two by two, and then by nine: this gives thirty-six; take the root of this, it is six, and this is double the root of nine.

\[
\begin{align*}
* \quad & 2\sqrt{x^2} = \sqrt{4x^2} \\
& 3\sqrt{x^2} = \sqrt{9x^2} \\
† \quad & n\sqrt{x^2} = \sqrt{n^2x^2} \\
‡ \quad & \frac{1}{2}\sqrt{x^2} = \sqrt{\frac{x^2}{4}} \\
§ \quad & \frac{1}{n}\sqrt{x^2} = \sqrt{\frac{x^2}{n^2}} \\
|| \quad & 2\sqrt{9} = \sqrt{4 \times 9} = \sqrt{36} = 6
\end{align*}
\]

In the same manner, if you require to triple the root of nine,* you multiply three by three, and then by nine: the product is eighty-one; take its root, it is nine, which becomes equal to thrice the root of nine.

If you require to have the moiety of the root of nine,† you multiply a half by a half, which gives a quarter, and then this by nine; the result is two and a quarter: take its root; it is one and a half, which is the moiety of the root of nine.

You proceed in this manner with every root, whether positive or negative, and whether known or unknown.

---

ON DIVISION.

If you will divide the root of nine by the root of four,‡ you begin with dividing nine by four, which gives two and a quarter: the root of this is the number which you require—it is one and a half.

If you will divide the root of four by the root of nine,§ you divide four by nine; it is four-ninths of the unit: the root of this is two divided by three; namely, two-thirds of the unit.

---

* $3\sqrt{9} = \sqrt{9 \times 9} = \sqrt{81} = 9$
† $\frac{1}{2}\sqrt{9} = \sqrt{\frac{9}{4}} = \sqrt{2\frac{1}{4}} = 1\frac{1}{2}$
‡ $\frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2} = \sqrt{2\frac{1}{4}} = 1\frac{1}{2}$
§ $\frac{\sqrt{4}}{\sqrt{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$
If you wish to divide twice the root of nine by the root of four, or of any other square*, you double the root of nine in the manner above shown to you in the chapter on Multiplication, and you divide the product by four, or by any number whatever. You perform this in the way above pointed out.

In like manner, if you wish to divide three roots of nine, or more, or one-half or any multiple or sub-multiple of the root of nine, the rule is always the same:† follow it, the result will be right.

If you wish to multiply the root of nine by the root of four,‡ multiply nine by four; this gives thirty-six; take its root, it is six; this is the root of nine, multiplied by the root of four.

Thus, if you wish to multiply the root of five by the root of ten,§ multiply five by ten: the root of the product is what you have required.

If you wish to multiply the root of one-third by the root of a half,|| you multiply one-third by a half: it is one-sixth: the root of one-sixth is equal to the root of one-third, multiplied by the root of a half.

If you require to multiply twice the root of nine by

\[
\begin{align*}
* \frac{2\sqrt{9}}{\sqrt{4}} &= \sqrt{\frac{36}{4}} = \sqrt{9} = 3 \\
† \frac{m\sqrt{p^2}}{\sqrt{q^2}} &= \sqrt{\frac{m^2p^2}{q^2}} \\
‡ \sqrt{4 \times 9} &= \sqrt{4 \times 9} = \sqrt{36} = 6 \\
§ \sqrt{10 \times 5} &= \sqrt{5 \times 10} = \sqrt{50} \\
|| \sqrt{\frac{1}{2} \times \frac{1}{3}} &= \sqrt{\frac{1}{2} \times \frac{1}{3}} = \sqrt{\frac{1}{6}}
\end{align*}
\]
thrice the root of four, * then take twice the root of nine, according to the rule above given, so that you may know the root of what square it is. You do the same with respect to the three roots of four in order to know what must be the square of such a root. You then multiply these two squares, the one by the other, and the root of the product is equal to twice the root of nine, multiplied by thrice the root of four.

You proceed in this manner with all positive or negative roots.

_Demonstrations._

The argument for the root of two hundred, minus ten, added to twenty, minus the root of two hundred, may be elucidated by a figure:

Let the line A B represent the root of two hundred; let the part from A to the point C be the ten, then the remainder of the root of two hundred will correspond to the remainder of the line A B, namely to the line C B. Draw now from the point B a line to the point D, to represent twenty; let it, therefore, be twice as long as the line A C, which represents ten; and mark a part of it from the point B to the point H, to be equal to the line A B, which represents the root of two hundred; then the remainder of the twenty will be equal to the part of the line, from the point H to the point D. As

\[
3\sqrt{4} \times 2\sqrt{9} = \sqrt{9 \times 4} \times \sqrt{4 \times 9} = \sqrt{36 \times 36} = 36
\]
our object was to add the remainder of the root of two hundred, after the subtraction of ten, that is to say, the line C B, to the line H D, or to twenty, minus the root of two hundred, we cut off from the line B H a piece equal to C B, namely, the line S H. We know already that the line A B, or the root of two hundred, is equal to the line B H, and that the line A C, which represents the ten, is equal to the line S B, as also that the remainder of the line A B, namely, the line C B is equal to the remainder of the line B H, namely, to S H. Let us add, therefore, this piece S H, to the line H D. We have already seen that from the line B D, or twenty, a piece equal to A C, which is ten, was cut off, namely, the piece B S. There remains after this the line S D, which, consequently, is equal to ten. This it was that we intended to elucidate. Here follows the figure.

The argument for the root of two hundred, minus ten, to be subtracted from twenty, minus the root of two hundred, is as follows. Let the line A B represent the root of two hundred, and let the part thereof, from A to the point C, signify the ten mentioned in the instance. We draw now from the point B, a line towards the point D, to signify twenty. Then we trace from B to the
point H, the same length as the length of the line which represents the root of two hundred; that is of the line A B. We have seen that the line C B is the remainder from the twenty, after the root of two hundred has been subtracted. It is our purpose, therefore, to subtract the line C B from the line H D; and we now draw from the point B, a line towards the point S, equal in length to the line A C, which represents the ten. Then the whole line S D is equal to S B, plus B D, and we perceive that all this added together amounts to thirty. We now cut off from the line H D, a piece equal to C B, namely, the line H G; thus we find that the line G D is the remainder from the line S D, which signifies thirty. We see also that the line B H is the root of two hundred and that the line S B and B C is likewise the root of two hundred. Now the line H G is equal to C B; therefore the piece subtracted from the line S D, which represents thirty, is equal to twice the root of two hundred, or once the root of eight hundred. (24) This it is that we wished to elucidate.

Here follows the figure:

As for the hundred and square minus twenty roots added to fifty, and ten roots minus two squares, this does
not admit of any figure, because there are three different species, *viz.* squares, and roots, and numbers, and nothing corresponding to them by which they might be represented. We had, indeed, contrived to construct a figure also for this case, but it was not sufficiently clear.

The elucidation by words is very easy. You know that you have a hundred and a square, minus twenty roots. When you add to this fifty and ten roots, it becomes a hundred and fifty and a square, minus ten roots. The reason for these ten negative roots is, that from the twenty negative roots ten positive roots were subtracted by reduction. This being done, there remains a hundred and fifty and a square, minus ten roots. With the hundred a square is connected. If you subtract from this hundred and square the two squares negative connected with fifty, then one square disappears by reason of the other, and the remainder is a hundred and fifty, minus a square, and minus ten roots.

This it was that we wished to explain.
OF THE SIX PROBLEMS.

Before the chapters on computation and the several (25) species thereof, I shall now introduce six problems, as instances of the six cases treated of in the beginning of this work. I have shown that three among these cases, in order to be solved, do not require that the roots be halved, and I have also mentioned that the calculating by completion and reduction must always necessarily lead you to one of these cases. I now subjoin these problems, which will serve to bring the subject nearer to the understanding, to render its comprehension easier, and to make the arguments more perspicuous.

First Problem.

I have divided ten into two portions; I have multiplied the one of the two portions by the other; after this I have multiplied the one of the two by itself, and the product of the multiplication by itself is four times as much as that of one of the portions by the other.*

Computation: Suppose one of the portions to be thing, and the other ten minus thing: you multiply

\[
\begin{align*}
x^2 &= 4x(10-x) = 40x - 4x^2 \\
x^2 &= 40x \\
x &= 8x \\
x &= 8; (10-x) = 2
\end{align*}
\]
thing by ten minus thing; it is ten things minus a square. Then multiply it by four, because the instance states "four times as much." The result will be four times the product of one of the parts multiplied by the other. This is forty things minus four squares. After this you multiply thing by thing, that is to say, one of the portions by itself. This is a square, which is equal to forty things minus four squares. Reduce it now by the four squares, and add them to the one square. Then the equation is: forty things are equal to five squares; and one square will be equal to eight roots, that is, sixty-four; the root of this is eight, and this is one of the two portions, namely, that which is to (26) be multiplied by itself. The remainder from the ten is two, and that is the other portion. Thus the question leads you to one of the six cases, namely, that of "squares equal to roots." Remark this.

Second Problem.

I have divided ten into two portions: I have multiplied each of the parts by itself, and afterwards ten by itself: the product of ten by itself is equal to one of the two parts multiplied by itself, and afterwards by two and seven-ninths; or equal to the other multiplied by itself, and afterwards by six and one-fourth.*

\[
\begin{align*}
10^2 &= x^2 \times \frac{27}{9} \\
100 &= x^2 \times \frac{25}{9} \\
\frac{9}{25} \times 100 &= x^2 \\
36 &= x^2 \\
6 &= x
\end{align*}
\]
Computation: Suppose one of the parts to be thing, and the other ten minus thing. You multiply thing by itself, it is a square; then by two and seven-ninths, this makes it two squares and seven-ninths of a square. You afterwards multiply ten by ten; it is a hundred, which much be equal to two squares and seven-ninths of a square. Reduce it to one square, through division by nine twenty-fifths;* this being its fifth and four-fifths of its fifth, take now also the fifth and four-fifths of the fifth of a hundred; this is thirty-six, which is equal to one square. Take its root, it is six. This is one of the two portions; and accordingly the other is four. This question leads you, therefore, to one of the six cases, namely, "squares equal to numbers."

Third Problem.

I have divided ten into two parts. I have afterwards divided the one by the other, and the quotient was four.†

Computation: Suppose one of the two parts to be (27) thing; the other ten minus thing. Then you divide ten minus thing by thing, in order that four may be obtained. You know that if you multiply the quotient by the divisor, the sum which was divided is restored.

\[ \frac{9}{25} = \frac{1}{5} \times \frac{4}{3} + \frac{1}{3} \]
\[ \frac{10-x}{x} = 4 \]
\[ 10-x = 4x \]
\[ 10 = 5x \]
\[ 2 = x \]
In the present question the quotient is four and the divisor is thing. Multiply, therefore, four by thing; the result is four things, which are equal to the sum to be divided, which was ten minus thing. You now reduce it by thing, which you add to the four things. Then we have five things equal to ten; therefore one thing is equal to two, and this is one of the two portions. This question refers you to one of the six cases, namely, "roots equal to numbers."

**Fourth Problem.**

I have multiplied one-third of thing and one dirhem by one-fourth of thing and one dirhem, and the product was twenty.*

Computation: You multiply one-third of thing by one-fourth of thing; it is one-half of a sixth of a square. Further, you multiply one dirhem by one-third of thing, it is one-third of thing; and one dirhem by one-fourth of thing, it is one-fourth of thing; and one dirhem by one dirhem, it is one dirhem. The result of this is: the moiety of one-sixth of a square, and one-third of thing, and one-fourth of thing, and one dirhem, is equal to twenty dirhems. Subtract now the one dirhem from

\[
\frac{1}{2} (\frac{1}{3} x + 1) \left( \frac{1}{4} x + 1 \right) = 20
\]

\[
\frac{x^2}{12} + \frac{1}{3} x + \frac{1}{4} x + 1 = 20
\]

\[
\frac{x^2}{12} + \frac{1}{2} x^2 = 19
\]

\[
x^2 + 7x = 228
\]

\[
x = \sqrt{\frac{10}{3} + 228} - \frac{7}{2} = 12
\]
these twenty dirhems, there remain nineteen dirhems, equal to the moiety of one-sixth of a square, and one-third of thing, and one-fourth of thing. Now make your square a whole one: you perform this by multiplying all that you have by twelve. Thus you have one square and seven roots, equal to two hundred and twenty-eight dirhems. Halve the number of the roots, and multiply it by itself; it is twelve and one-fourth. Add this to the numbers, that is, to two hundred and twenty-eight; (28) the sum is two hundred and forty and one quarter. Extract the root of this; it is fifteen and a half. Subtract from this the moiety of the roots, that is, three and a half, there remains twelve, which is the square required. This question leads you to one of the cases, namely, "squares and roots equal to numbers."

**Fifth Problem.**

I have divided ten into two parts; I have then multiplied each of them by itself, and when I had added the products together, the sum was fifty-eight dirhems.*

Computation: Suppose one of the two parts to be thing, and the other ten minus thing. Multiply ten minus thing by itself; it is a hundred and a square minus twenty things. Then multiply thing by thing; it

\[
* x^2 + (10 - x)^2 = 58 \\
2x^2 - 20x + 100 = 58 \\
x^2 - 10x + 50 = 29 \\
x^2 + 21 = 10x \\
x = 5 \pm \sqrt{25 - 21} = 5 \pm 2 = 7 \text{ or } 3
\]
is a square. Add both together. The sum is a hundred, plus two squares minus twenty things, which are equal to fifty-eight dirhems. Take now the twenty negative things from the hundred and the two squares, and add them to fifty-eight; then a hundred, plus two squares, are equal to fifty-eight dirhems and twenty things. Reduce this to one square, by taking the moiety of all you have. It is then: fifty dirhems and a square, which are equal to twenty-nine dirhems and ten things. Then reduce this, by taking twenty-nine from fifty; there remains twenty-one and a square, equal to ten things. Halve the number of the roots, it is five; multiply this by itself, it is twenty-five; take from this the twenty-one which are connected with the square, the remainder is four. Extract the root, it is two. Subtract this from the moiety of the roots, namely, from five, there remains three. This is one of the portions; the other is seven. This question refers you to one of the six cases, namely "squares and numbers equal to roots."

Sixth Problem.

I have multiplied one-third of a root by one-fourth of a root, and the product is equal to the root and twenty-four dirhems.*

\[
\begin{align*}
* \quad & \frac{x}{3} \times \frac{x}{4} = x + 24 \\
& \frac{x^3}{12} = x + 24 \\
& x^2 = 12x + 288 \\
& x = 6 + \sqrt{36 + 288} = 6 + 18 = 24
\end{align*}
\]
Computation: Call the root thing; then one-third of thing is multiplied by one-fourth of thing; this is the moiety of one-sixth of the square, and is equal to thing and twenty-four dirhems. Multiply this moiety of one-sixth of the square by twelve, in order to make your square a whole one, and multiply also the thing by twelve, which yields twelve things; and also four-and-twenty by twelve: the product of the whole will be two hundred and eighty-eight dirhems and twelve roots, which are equal to one square. The moiety of the roots is six. Multiply this by itself, and add it to two hundred and eighty-eight, it will be three hundred and twenty-four. Extract the root from this, it is eighteen; add this to the moiety of the roots, which was six; the sum is twenty-four, and this is the square sought for. This question refers you to one of the six cases, namely, "roots and numbers equal to squares."

VARIOUS QUESTIONS.

If a person puts such a question to you as: "I have divided ten into two parts, and multiplying one of these by the other, the result was twenty-one;"* then

\[
* \quad (10 - x)x = 21 \\
10x - x^2 = 21 \\
\text{which is to be reduced to} \\
x^2 + 21 = 10x \\
x = 5 \pm \sqrt{25 - 21} = 5 \pm 2
\]
you know that one of the two parts is thing, and the other ten minus thing. Multiply, therefore, thing by ten minus thing; then you have ten things minus a square, which is equal to twenty-one. Separate the square from the ten things, and add it to the twenty-one. Then you have ten things, which are equal to twenty-one dirhems and a square. Take away the moiety of the roots, and multiply the remaining five by itself; it is twenty-five. Subtract from this the twenty-one which are connected with the square; the remainder is four. Extract its root, it is two. Subtract this from the moiety of the roots, namely, five; there remain three, which is one of the two parts. Or, if you please, you may add the root of four to the moiety of the roots; the sum is seven, which is likewise one of the parts. This is one of the problems which may be resolved by addition and subtraction.

If the question be: "I have divided ten into two parts, and having multiplied each part by itself, I have subtracted the smaller from the greater, and the remainder was forty;"* then the computation is—you multiply ten minus thing by itself, it is a hundred plus one square minus twenty things; and you also multiply thing by

\[
\begin{align*}
(10 - x)^2 - x^2 &= 40 \\
100 - 20x &= 40 \\
100 &= 20x + 40 \\
60 &= 20x \\
3 &= x
\end{align*}
\]
thing, it is one square. Subtract this from a hundred and a square minus twenty things, and you have a hundred, minus twenty things, equal to forty dirhems. Separate now the twenty things from a hundred, and add them to the forty; then you have a hundred, equal to twenty things and forty dirhems. Subtract now forty from a hundred; there remains sixty dirhems, equal to twenty things: therefore one thing is equal to three, which is one of the two parts.

If the question be: "I have divided ten into two parts, and having multiplied each part by itself, I have put them together, and have added to them the difference of the two parts previously to their multiplication, and the amount of all this is fifty-four;"* then the computation is this: You multiply ten minus thing by itself; it is a hundred and a square minus twenty things. Then multiply also the other thing of the ten by itself; it is one square. Add this together, it will be a hundred plus two squares minus twenty things. It was stated that the difference of the two parts before multiplication should be added to them. You say, therefore, the difference between them is ten minus two things.

\[
\begin{align*}
(10 - x)^2 + x^2 + (10 - x) - x &= 54 \\
100 - 20x + 2x^2 + 10 - 2x &= 54 \\
100 - 22x + 2x^2 &= 54 \\
55 - 11x + x^2 &= 27 \\
x^2 + 28 &= 11x
\end{align*}
\]

\[
x = \frac{11}{2} \pm \sqrt{\frac{121}{4} - 28} = \frac{11 \pm 3}{2} = 7 \text{ or } 4
\]
The result is a hundred and ten and two squares minus twenty-two things, which are equal to fifty-four dirhems. Having reduced and equalized this, you may say, a hundred and ten dirhems and two squares are equal to fifty-four dirhems and twenty-two things. Reduce now the two squares to one square, by taking the moiety of all you have. Thus it becomes fifty-five dirhems and a square, equal to twenty-seven dirhems and eleven things. Subtract twenty-seven from fifty-five, there remain twenty-eight dirhems and a square, equal to eleven things. Halve now the things, it will be five and a half; multiply this by itself, it is thirty and a quarter. Subtract from it the twenty-eight which are combined with the square, the remainder is two and a fourth. Extract its root, it is one and a half. Subtract this from the moiety of the roots, there remain four, which is one of the two parts.

If one say, "I have divided ten into two parts; and have divided the first by the second, and the second by the first, and the sum of the quotient is two dirhems and one-sixth;"* then the computation is this: If you multiply each part by itself, and add the products together, then their sum is equal to one of the parts

\[
\begin{align*}
\frac{10-x}{x} + \frac{x}{10-x} &= 2^\frac{1}{6} \\
100 + 2x^2 - 20x &= x(10-x) \times 2^\frac{1}{6} = 21^\frac{3}{6}x - 2^\frac{1}{6}x^2 \\
100 + 4^\frac{1}{6}x^2 &= 41^\frac{3}{6}x \\
24 + x^2 &= 10x \\
x &= 5 \pm \sqrt{25-24} = 5 \pm 1 = 4 \text{ or } 6
\end{align*}
\]
multiplied by the other, and again by the quotient which is two and one-sixth. Multiply, therefore, ten less thing by itself; it is a hundred and a square less ten things. Multiply thing by thing; it is one square. Add this together; the sum is a hundred plus two squares less twenty things, which is equal to thing multiplied by ten less thing; that is, to ten things less a square, multiplied by the sum of the quotients arising from the division of the two parts, namely, two and one-sixth. We have, therefore, twenty-one things and two-thirds of thing less two squares and one-sixth, equal to a hundred plus two squares less twenty things. Reduce this by adding the two squares and one-sixth to a hundred plus two squares less twenty things, and add the twenty negative things from the hundred plus the two squares to the twenty-one things and two-thirds of thing. Then you have a hundred plus four squares and one-sixth of a square, equal to forty-one things and two-thirds of thing. Now reduce this to one square. You know that one square is obtained from four squares and one-sixth, by taking a fifth and one-fifth of a fifth.* Take, therefore, the fifth and one-fifth of a fifth of all that you have. Then it is twenty-four and a square, equal to ten roots; because ten is one-fifth and one-fifth of the fifth of the forty-one things and two-thirds of a thing. Now halve the roots; it gives five. Multiply this

\[ 4\frac{1}{6} = \frac{25}{6} \text{ and } \frac{5}{23} = \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \]
by itself; it is five-and-twenty. Subtract from this the twenty-four, which are connected with the square; the remainder is one. Extract its root; it is one. Subtract this from the moiety of the roots, which is five. There remains four, which is one of the two parts.

Observe that, in every case, where any two quantities whatsoever are divided, the first by the second and the second by the first, if you multiply the quotient of the one division by that of the other, the product is always one.*

If some one say: "You divide ten into two parts; multiply one of the two parts by five, and divide it by the other: then take the moiety of the quotient, and add this to the product of the one part, multiplied by five; the sum is fifty dirhems;"† then the computation is this: Take thing, and multiply it by five. This is now to be divided by the remainder of the ten, that is, by ten less thing; and of the quotient the moiety is to be taken.

(34) You know that if you divide five things by ten less thing, and take the moiety of the quotient, the result is

\[
\begin{align*}
* \quad \frac{a}{b} \times \frac{b}{a} &= 1 \\
\uparrow \quad \frac{5x}{2(10-x)} + 5x &= 50 \\
& \quad \frac{x}{2(10-x)} + x = 10 \\
& \quad x^2 + 100 = 20 \frac{1}{2}x \\
& \quad x = 4 \frac{1}{4} - \frac{9}{4} = 8
\end{align*}
\]
the same as if you divide the moiety of five things by ten less thing. Take, therefore, the moiety of five things; it is two things and a half: and this you require to divide by ten less thing. Now these two things and a half, divided by ten less thing, give a quotient which is equal to fifty less five things: for the question states: add this (the quotient) to the one part multiplied by five, the sum will be fifty. You have already observed, that if the quotient, or the result of the division, be multiplied by the divisor, the dividend, or capital to be divided, is restored. Now, your capital, in the present instance, is two things and a half. Multiply, therefore, ten less thing by fifty less five things. Then you have five hundred dirhems and five squares less a hundred things, which are equal to two things and a half. Reduce this to one square. Then it becomes a hundred dirhems and a square less twenty things, equal to the moiety of thing. Separate now the twenty things from the hundred dirhems and square, and add them to the half thing. Then you have a hundred dirhems and a square, equal to twenty things and a half. Now halve the things, multiply the moiety by itself, subtract from this the hundred, extract the root of the remainder, and subtract this from the moiety of the roots, which is ten and one-fourth: the remainder is eight; and this is one of the portions.

If some one say: "You divide ten into two parts: multiply the one by itself; it will be equal to the other
taken eighty-one times."* Computation: You say, ten
less thing, multiplied by itself, is a hundred plus a
(35) square less twenty things, and this is equal to eighty-
one things. Separate the twenty things from a hundred
and a square, and add them to eighty-one. It will
then be a hundred plus a square, which is equal to a
hundred and one roots. Halve the roots; the moiety is
fifty and a half. Multiply this by itself, it is two thou-
sand five hundred and fifty and a quarter. Subtract
from this one hundred; the remainder is two thousand
four hundred and fifty and a quarter. Extract the root
from this; it is forty-nine and a half. Subtract this
from the moiety of the roots, which is fifty and a half.
There remains one, and this is one of the two parts.

If some one say: "I have purchased two measures of
wheat or barley, each of them at a certain price. I
afterwards added the expenses, and the sum was equal
to the difference of the two prices, added to the diffe-
rence of the measures."†

\[
\begin{align*}
\ast (10 - x)^2 &= 81x \\
100 - 20x + x^2 &= 81x \\
x^2 + 100 &= 101x \\
x &= \frac{101}{2} - \sqrt{\frac{101^2}{4} - 100} = 50\frac{1}{2} - 49\frac{1}{2} = 1
\end{align*}
\]

† The purchaser does not make a clear enunciation of the
terms of his bargain. He intends to say, "I bought \(m\)
bushels of wheat, and \(n\) bushels of barley, and the wheat was\(r\) times dearer than the barley. The sum I expended was
equal to the difference in the quantities, added to the diffe-
rence in the prices of the grain."
Computation: Take what numbers you please, for it is indifferent; for instance, four and six. Then you say: I have bought each measure of the four for thing; and accordingly you multiply four by thing, which gives four things; and I have bought the six, each for the moiety of thing, for which I have bought the four; or, if you please, for one-third, or one-fourth, or for any other quota of that price, for it is indifferent. Suppose that you have bought the six measures for the moiety of thing, then you multiply the moiety of thing by six; this gives three things. Add them to the four things; the sum is seven things, which must be equal to the difference of the two quantities, which is two measures, plus the difference of the two prices, which is a moiety of thing. You have, therefore, seven things, equal to two and a moiety of thing. Remove, now, this moiety of thing, by subtracting it from the seven things. There remain six things and a half, equal to two dirhems: consequently, one thing is equal to four-thirteenth of a dirhem. The six measures were bought, each at one-half of thing; that is, at two-thirteenth of a dirhem. Accordingly, the expenses amount to eight-and-twenty thirteenth of a dirhem, and this sum is equal to the difference of the two quantities; namely,

If \( x \) is the price of the barley, \( rx \) is the price of the wheat; whence, \( mrx + nx = (m - n) + (rx - x) \); \( x = \frac{m-n}{mr+n+r} \) and the sum expended is \( \frac{(mr+n)x(m-n)}{mr+n+r} \).
the two measures, the arithmetical equivalent for which is six-and-twenty thirteenths, added to the difference of the two prices, which is two-thirteenths: both differences together being likewise equal to twenty-eight parts.

If he say: "There are two numbers,* the difference of which is two dirhems. I have divided the smaller by the larger, and the quotient was just half a dirhem."†

Suppose one of the two numbers* to be thing, and the other to be thing plus two dirhems. By the division of thing by thing plus two dirhems, half a dirhem appears as quotient. You have already observed, that by multiplying the quotient by the divisor, the capital which you divided is restored. This capital, in the present case, is thing. Multiply, therefore, thing and two dirhems by half a dirhem, which is the quotient; the product is half one thing plus one dirhem; this is equal to thing. Remove, now, half a thing on account

* In the original, " squares." The word square is used in the text to signify either, 1st, a square, properly so called, fractional or integral; 2d, a rational integer, not being a square number; 3d, a rational fraction, not being a square; 4th, a quadratic surd, fractional or integral.

† \[
\begin{align*}
\frac{x}{x + 2} &= \frac{1}{2} \\
x &= \frac{x + 2}{2} = \frac{x}{2} + 1 \\
\frac{x}{2} &= 1 \text{ and } x + 2 = 4
\end{align*}
\]
of the other half thing; there remains one dirhem, equal to half a thing. Double it, then you have one thing, equal to two dirhems. Consequently, the other number* is four.

If some one say: “I have divided ten into two parts; I have multiplied the one by ten and the other by itself, and the products were the same;”† then the computation is this: You multiply thing by ten; it is ten things. Then multiply ten less thing by itself; it is a hundred (37) and a square less twenty things, which is equal to ten things. Reduce this according to the rules, which I have above explained to you.

In like manner, if he say: “I have divided ten into two parts; I have multiplied one of the two by the other, and have then divided the product by the difference of the two parts before their multiplication, and the result of this division is five and one-fourth;”‡ the computation will be this: You subtract thing from ten; there remain ten less thing. Multiply the one by the other, it is ten things less a square. This is the product of the multiplication of one of the two parts by the other. At

* "Square" in the original.

† \[10x = (10 - x)^2 = 100 - 20x + x^2\]
\[x = 15 - \sqrt{225 - 100} = 15 - \sqrt{125}\]
\[\frac{x(10 - x)}{10 - 2x} = 5\frac{1}{4}\]
\[10x - x^2 = 5\frac{3}{2} - 10\frac{1}{4}x\]
\[20\frac{1}{2}x = x^2 + 5\frac{1}{2}\]
\[x = 10\frac{1}{2} - 7\frac{1}{2} = 3\]
present you divide this by the difference between the two parts, which is ten less two things. The quotient of this division is, according to the statement, five and a fourth. If, therefore, you multiply five and one-fourth by ten less two things, the product must be equal to the above amount, obtained by multiplication, namely, ten things less one square. Multiply now five and one-fourth by ten less two squares. The result is fifty-two dirhems and a half less ten roots and a half, which is equal to ten roots less a square. Separate now the ten roots and a half from the fifty-two dirhems, and add them to the ten roots less a square; at the same time separate this square from them, and add it to the fifty-two dirhems and a half. Thus you find twenty roots and a half, equal to fifty-two dirhems and a half and one square. Now continue reducing it, conformably to the rules explained at the commencement of this book.

(38) If the question be: "There is a square, two-thirds of one-fifth of which are equal to one-seventh of its root;" then the square is equal to one root and half a seventh of a root; and the root consists of fourteen-fifteenths of the square.* The computation is this: You

\[
\frac{2}{3} \times \frac{1}{3} x^2 = \frac{x}{7}
\]

\[
x^2 = 7\frac{1}{2} \times \frac{x}{7} = 1\frac{1}{4} x
\]

\[
x = 1\frac{1}{4}
\]

\[
x^2 = 1\frac{9}{6}
\]

\[
\frac{2}{15} x^2 = \frac{3}{196} \times \frac{x}{7}
\]
multiply two-thirds of one-fifth of the square by seven and a half, in order that the square may be completed. Multiply that which you have already, namely, one-seventh of its root, by the same. The result will be, that the square is equal to one root and half a seventh of the root; and the root of the square is one and a half seventh; and the square is one and twenty-nine one hundred and ninety-sixths of a dirhem. Two-thirds of the fifth of this are thirty parts of the hundred and ninety-six parts. One-seventh of its root is likewise thirty parts of a hundred and ninety-six.

If the instance be: "Three-fourths of the fifth of a square are equal to four-fifths of its root,"* then the computation is this: You add one-fifth to the four-fifths, in order to complete the root. This is then equal to three and three-fourths of twenty parts, that is, to fifteen eightieths of the square. Divide now eighty by fifteen; the quotient is five and one-third. This is the root of the square, and the square is twenty-eight and four-ninths.

If some one say: "What is the amount of a square-root,† which, when multiplied by four times itself,

* \[ \frac{3}{4} \times \frac{1}{5}x^2 = \frac{3}{5}x \]

\[ \frac{3\frac{3}{4}}{20}x, \text{ or } \frac{1}{5}x, \text{ or } \frac{3}{10}x = 1 \]

\[ \therefore x = \frac{10}{3} = 5\frac{1}{3} \]

† "Square" in the original.
amounts to twenty?*" the answer is this: If you multiply it by itself it will be five: it is therefore the root of five.

If somebody ask you for the amount of a square-root,† which when multiplied by its third amounts to ten,‡ the solution is, that when multiplied by itself it will amount to thirty; and it is consequently the root of thirty.

(39) If the question be: "To find a quantity†, which when multiplied by four times itself, gives one-third of the first quantity as product,"§ the solution is, that if you multiply it by twelve times itself, the quantity itself must re-appear: it is the moiety of one moiety of one-third.

If the question be: "A square, which when multiplied by its root gives three times the original square as product,"¶ then the solution is: that if you multiply the root by one-third of the square, the original square is

\[
\begin{align*}
\ast & \quad 4x^2 = 20 \\
& \quad x = \sqrt{5} \\
† & \quad "\text{Square}" \text{ in the original.} \\
‡ & \quad x \times \frac{x}{3} = 10 \\
& \quad x^2 = 30 \\
& \quad x = \sqrt{30} \\
§ & \quad x \times 4x = \frac{x}{3} \\
& \quad x = \frac{1}{\sqrt{3}} \\
¶ & \quad x^2 \times x = 3x^2 \\
& \quad x = 3
\end{align*}
\]
restored; its root must consequently be three, and the square itself nine.

If the instance be: "To find a square, four roots of which, multiplied by three roots, restore the square with a surplus of forty-four dirhems,"* then the solution is: that you multiply four roots by three roots, which gives twelve squares, equal to a square and forty-four dirhems. Remove now one square of the twelve on account of the one square connected with the forty-four dirhems. There remain eleven squares, equal to forty-four dirhems. Make the division, the result will be four, and this is the square.

If the instance be: "A square, four of the roots of which multiplied by five of its roots produce twice the square, with a surplus of thirty-six dirhems;"† then the solution is: that you multiply four roots by five roots, which gives twenty squares, equal to two squares and thirty-six dirhems. Remove two squares from the twenty on account of the other two. The remainder is eighteen squares, equal to thirty-six dirhems. Divide now thirty-six dirhems by eighteen; the quotient is two, and this is the square.

* \(4x \times 3x = x^2 + 44\)

\[11x^2 = 44\]

\[x^2 = 4\]

\[x = 2\]

† \(4x \times 5x = 2x^2 + 36\)

\[18x^2 = 36\]

\[x^2 = 2\]
In the same manner, if the question be: "A square, multiply its root by four of its roots, and the product will be three times the square, with a surplus of fifty dirhems."† Computation: You multiply the root by four roots, it is four squares, which are equal to three squares and fifty dirhems. Remove three squares from the four; there remains one square, equal to fifty dirhems. One root of fifty, multiplied by four roots of the same, gives two hundred, which is equal to three times the square, and a residue of fifty dirhems.

If the instance be: "A square, which when added to twenty dirhems, is equal to twelve of its roots,"† then the solution is this: You say, one square and twenty dirhems are equal to twelve roots. Halve the roots and multiply them by themselves; this gives thirty-six. Subtract from this the twenty dirhems, extract the root from the remainder, and subtract it from the moiety of the roots, which is six. The remainder is the root of the square: it is two dirhems, and the square is four.

If the instance be: "To find a square, of which if one-third be added to three dirhems, and the sum be subtracted from the square, the remainder multiplied by

\[ 4x^2 = 3x^2 + 50 \]
\[ x^2 = 50 \]
\[ x^2 + 20 = 12x \]
\[ x = 6 \pm \sqrt{36 - 20} = 6 \pm 4 = 10 \text{ or } 2 \]
itself restores the square;"* then the computation is this: If you subtract one-third and three dirhems from the square, there remain two-thirds of it less three dirhems. This is the root. Multiply therefore two-thirds of thing less three dirhems by itself. You say two-thirds by two-thirds is four ninths of a square; and less two-thirds by three dirhems is two roots: and again, two-thirds by three dirhems is two roots; and less three dirhems by less three dirhems is nine dirhems. You have, therefore, four-ninths of a square and nine dirhems less four roots, which are equal to one root. Add the four roots to the one root, then you have five roots, which are equal to four-ninths of a square and nine dirhems. Complete now your square; that is, multiply the four-ninths of a square by two and a fourth, which gives one square; multiply likewise the nine dirhems by two and a quarter; this gives twenty and a quarter; finally, multiply the five roots by two and a quarter; this gives eleven roots and a quarter. You have, therefore, a square and twenty dirhems and a quarter, equal to eleven roots and a quarter. Reduce this according to what I taught you about halving the roots.

\[* [x-(\frac{x}{3}+3)]^2=x\]

or \[^{\frac{2x}{3}}-3]^2=x\]

\[\frac{4x^2}{9}+9=5x\]

\[x^2+20=11\frac{1}{4}x\]

\[x=9, \text{ or } 2\frac{1}{4}\]
If the instance be: "To find a number,* one-third of which, when multiplied by one-fourth of it, restores the *number,"† then the computation is: You multiply one-third of thing by one-fourth of thing, this gives one-twelfth of a square, equal to thing, and the square is equal to twelve things, which is the root of one hundred and forty-four.

If the instance be: "A number,* one-third of which and one dirhem multiplied by one-fourth of it and two dirhems restore the number,* with a surplus of thirteen dirhems;"‡ then the computation is this: You multiply one-third of thing by one-fourth of thing, this gives half one-sixth of a square; and you multiply two dirhems by one-third of thing, this gives two-thirds of a root; and one dirhem by one-fourth of thing gives one-fourth of a root; and one dirhem by two dirhems gives two dirhems. This altogether is one-twelfth of a square and two dirhems and eleven-twelfths of a thing, equal to thing and thirteen dir-

* "Square" in the original.

† $\frac{x}{3} \times \frac{x}{4} = x$
$x^2 = 12x$
$x = 12$

‡ $\left(\frac{x}{3} + 1\right) - \left(\frac{x}{4} + 2\right) = x + 13$
$\frac{x^2}{12} + \frac{11}{12}x + 2 = x + 13$
$\frac{x^2}{12} = \frac{x}{12} + \frac{11}{2}$
$x^2 = x + 132$
$x = \frac{1}{2} + \frac{23}{2} = 12$
hems. Remove now two dirhems from thirteen, on account of the other two dirhems, the remainder is eleven dirhems. Remove then the eleven-twelfths of a root from the one (root on the opposite side), there remains one-twelfth of a root and eleven dirhems, equal to one-twelfth of a square. Complete the square: that is, multiply it by twelve, and do the same with all you have. The product is a square, which is equal to a hundred and thirty-two dirhems and one root. Reduce this, according to what I have taught you, it will be right.

If the instance be: “A dirhem and a half to be divided among one person and certain persons, so that the share of the one person be twice as many dirhems as there are other persons;”* then the Computation is this:† You say, the one person and some persons are one and thing: it is the same as if the question had been one dirhem and a half to be divided by one and thing, and the share of one person to be equal to two things. Multiply, therefore, two things by one and

* The enunciation in the original is faulty, and I have altered it to correspond with the computation. But in the computation, \( x \), the number of persons, is fractional! I am unable to correct the passage satisfactorily.

\[
\begin{align*}
\frac{1\frac{1}{2}}{1+x} &= 2x \\
x^2 + x &= \frac{3}{4} \\
x &= 1 - \frac{1}{2} \\
x &= \frac{1}{2}
\end{align*}
\]
thing; it is two squares and two things, equal to one dirhem and a half. Reduce them to one square: that is, take the moiety of all you have. You say, therefore, one square and one thing are equal to three-fourths of a dirhem. Reduce this, according to what I have taught you in the beginning of this work.

If the instance be: “A number,* you remove one-third of it, and one-fourth of it, and four dirhems: then you multiply the remainder by itself, and the number,* is restored, with a surplus of twelve dirhems:”† then the computation is this: You take thing, and subtract from it one-third and one-fourth; there remain five-twelfths of thing. Subtract from this four dirhems:

(43) the remainder is five-twelfths of thing less four dirhems. Multiply this by itself. Thus the five parts become five-and-twenty parts; and if you multiply twelve by itself, it is a hundred and forty-four. This makes, therefore, five and twenty hundred and forty-fourths of a square. Multiply then the four dirhems twice by the five-twelfths; this gives forty parts, every twelve of which make one root (forty-twelfths); finally, the four

* “Square” in the original.
† \( (x - \frac{1}{3}x - \frac{1}{4}x - 4)^2 = x + 12 \)
\( \left( \frac{5}{12}x - 4 \right)^2 = x + 12 \)
\( \frac{125}{144}x^2 + 16 - 3\frac{1}{3}x = x + 12 \)
\( \frac{25}{144}x^2 + 4 = 4\frac{1}{3}x \)
\( x^2 + 23\frac{1}{25} = 24\frac{4}{25}x \)
\( \sqrt{\left[ (\frac{24\frac{2}{5}}{2})^2 - 23\frac{1}{25} \right]} + \frac{24\frac{2}{5}}{2} - x \)
\( 11\frac{13}{25} + 12\frac{1}{5} = 24 = x \)
dirhems, multiplied by four dirhems, give sixteen dirhems to be added. The forty-twelfths are equal to three roots and one-third of a root, to be subtracted. The whole product is, therefore, twenty-five-hundred-and-forty-fourths of a square and sixteen dirhems less three roots and one-third of a root, equal to the original number,* which is thing and twelve dirhems. Reduce this, by adding the three roots and one-third to the thing and twelve dirhems. Thus you have four roots and one-third of a root and twelve dirhems. Go on balancing, and subtract the twelve (dirhems) from sixteen; there remain four dirhems and five-and-twenty-hundred-and-forty-fourths of a square, equal to four roots and one-third. Now it is necessary to complete the square. This you can accomplish by multiplying all you have by five and nineteen twenty-fifths. Multiply, therefore, the twenty-five-one-hundred-and-forty-fourths of a square by five and nineteen twenty-fifths. This gives a square. Then multiply the four (44) dirhems by five and nineteen twenty-fifths; this gives twenty-three dirhems and one twenty-fifth. Then multiply four roots and one-third by five and nineteen twenty-fifths; this gives twenty-four roots and twenty-four twenty-fifths of a root. Now halve the number of the roots: the moiety is twelve roots and twelve twenty-fifths of a root. Multiply this by itself. It is one hundred-and-fifty-five dirhems and four hundred-and-

* "Square" in the original.
sixty-nine six-hundred-and-twenty-fifths. Subtract from this the twenty-three dirhems and the one twenty-fifth connected with the square. The remainder is one-hundred-and-thirty-two and four-hundred-and-forty six-hundred-and-twenty-fifths. Take the root of this: it is eleven dirhems and thirteen twenty-fifths. Add this to the moiety of the roots, which was twelve dirhems and twelve twenty-fifths. The sum is twenty-four. It is the number* which you sought. When you subtract its third and its fourth and four dirhems, and multiply the remainder by itself, the number * is restored, with a surplus of twelve dirhems.

If the question be: “To find a square-root,* which, when multiplied by two-thirds of itself, amounts to five;” then the computation is this: You multiply one thing by two-thirds of thing; the product is two-thirds of square, equal to five. Complete it by adding its moiety to it, and add to five likewise its moiety. Thus you have a square, equal to seven and a half. Take its root; it is the thing which you required, and which, when multiplied by two-thirds of itself, is equal to five.

If the instance be: “Two numbers,† the difference

* “Square” in the original.
† $x \times \frac{2}{3}x = 5$
   $\frac{2}{3}x^2 = 5$
   $x^2 = 7\frac{1}{2}$
   $x = \sqrt{7\frac{1}{2}}$
‡ “Squares” in the original.
of which is two dirhems; you divide the small one by the great one, and the quotient is equal to half a dirhem;* then the computation is this: Multiply thing and two dirhems by the quotient, that is a half. The product is half a thing and one dirhem, equal to thing. Remove now half a dirhem on account of the half dirhem on the other side. The remainder is one dirhem, equal to half a thing. Double it: then you have thing, equal to two dirhems. This is one of the two numbers,† and the other is four.

Instance: "You divide one dirhem amongst a certain number of men, which number is thing. Now you add one man more to them, and divide again one dirhem amongst them; the quota of each is then one-sixth of a dirhem less than at the first time."‡ Computation: You multiply the first number of men, which is thing, by the difference of the share for each of the other number. Then multiply the product by the first and second number of men, and divide the product by the

* \( \frac{x}{x + 2} = \frac{1}{2} \)
\( \frac{1}{2}x + 1 = x \)
\( \frac{1}{2}x = 1 \)
\( x = 2, \ x + 2 = 4 \)

† "Squares" in the original.

‡ \( \frac{1}{x} - \frac{1}{x + 1} = \frac{1}{6} \)
\( \frac{5}{x} = \frac{x(x + 1)}{6} \)
\( x^2 + x = 6 \)
\( \sqrt{[\frac{1}{2}]^2 + 6} - \frac{1}{2} = x = 2 \)
difference of these two numbers. Thus you obtain the sum which shall be divided. Multiply, therefore, the first number of men, which is thing, by the one-sixth, which is the difference of the shares; this gives one-sixth of root. Then multiply this by the original number of the men, and that of the additional one, that is to say, by thing plus one. The product is one-sixth of square and one-sixth of root divided by one dirhem, and this is equal to one dirhem. Complete the square which you have through multiplying it by six. Then you have a square and a root equal to six dirhems. Halve the root and multiply the moiety by itself, it is one-fourth. Add this to the six; take the root of the sum and subtract from it the moiety of the root, which you have multiplied by itself, namely, a half. The remainder is the first number of men; which in this instance is two.

If the instance be: “To find a square-root,* which when multiplied by two-thirds of itself amounts to five;”† then the computation is this: If you multiply it by itself, it gives seven and a half. Say, therefore,

* “Square” in the original.
† $\frac{2}{3}x^2 = 5$
$x^2 = 7\frac{1}{2}$
$x = \sqrt{7\frac{1}{2}}$
$\sqrt{7\frac{1}{2}} \times \frac{2}{3}\sqrt{7\frac{1}{2}} = 5$
$\sqrt{\frac{3}{2}} \times 7\frac{1}{2} = \sqrt{3\frac{1}{2}} = \frac{\sqrt{3}}{2}$
$\sqrt{3\frac{1}{2}} \times 7\frac{1}{2} = \sqrt{25} = 5$
it is the root of seven and a half multiplied by two-thirds of the root of seven and a half. Multiply then two-thirds by two-thirds, it is four-ninths; and four-ninths multiplied by seven and a half is three and a third. The root of three and a third is two-thirds of the root of seven and a half. Multiply three and a third by seven and a half; the product is twenty-five, and its root is five.

If the instance be: "A square multiplied by three of its roots is equal to five times the original square,"* then this is the same as if it had been said, a square, which when multiplied by its root, is equal to the first square and two-thirds of it. Then the root of the square is one and two-thirds, and the square is two dirhems and seven-ninths.

If the instance be: "Remove one-third from a square, then multiply the remainder by three roots of the first square, and the first square will be restored."†

Computation: If you multiply the first square, before removing two-thirds from it, by three roots of the same, then it is one square and a half; for according to the statement two-thirds of it multiplied by three

\[
\begin{align*}
* \quad x^2 \times 3x &= 5x^2 \\
x^2 \times x &= 1\frac{2}{3}x^2 \\
x &= 1\frac{2}{3} \\
x^2 &= 2\frac{7}{9}
\end{align*}
\]

\[
\uparrow \quad (x^2 - \frac{1}{3}x^2) \times 3x = x^2 \quad \therefore \quad \frac{2}{3}x^2 \times 3x = x^2 \\
x^2 \times 3x = 1\frac{1}{3}x^2 \\
x = \frac{1}{2} \quad \therefore \quad x^2 = \frac{1}{4}
\]

K
roots give one square; and, consequently, the whole of it multiplied by three roots of it gives one square and a half. This entire square, when multiplied by one root, gives half a square; the root of the square must therefore be a half, the square one-fourth, two-thirds of the square one-sixth, and three roots of the square one and a half. If you multiply one-sixth by one and a half, the product is one-fourth, which is the square.

Instance: “A square; you subtract four roots of the same, then take one-third of the remainder; this is equal to the four roots.” The square is two hundred and fifty-six.* Computation: You know that one-third of the remainder is equal to four roots; consequently, the whole remainder must be twelve roots; add to this the four roots; the sum is sixteen, which is the root of the square.

Instance: “A square; you remove one root from it; and if you add to this root a root of the remainder, the sum is two dirhems.”† Then, this is the root of a

\[ \frac{x^2 - 4x}{3} = 4x \]
\[ x^2 - 4x = 12x \]
\[ x^2 = 16x \]
\[ x = 16 \therefore x^2 = 256 \]
\[ \sqrt{x^2 - x + x} = 2 \]
\[ \sqrt{x^2 - x} = 2 - x \]
\[ x^2 - x = 4 + x^2 - 4x \]
\[ x^2 + 3x = 4 + x^2 \]
\[ 3x = 4 \]
\[ x = 1\frac{1}{3} \]
square, which, when added to the root of the same square, less one root, is equal to two dirhems. Subtract from this one root of the square, and subtract also from the two dirhems one root of the square. Then two dirhems less one root multiplied by itself is four dirhems and one square less four roots, and this is equal to a square less one root. Reduce it, and you find a square and four dirhems, equal to a square and three roots. Remove square by square; there remain three roots, equal to four dirhems; consequently, one root is equal to one dirhem and one-third. This is the root of the square, and the square is one dirhem and seven-ninths of a dirhem.

Instance: "Subtract three roots from a square, then multiply the remainder by itself, and the square is restored."* You know by this statement that the remainder must be a root likewise; and that the square consists of four such roots; consequently, it must be sixteen.

\[ (x^2 - 3x)^2 = x^2 \]
\[ x^2 - 3x = x \]
\[ x^2 = 4x \]
\[ x = 4 \]
ON MERCANTILE TRANSACTIONS.

You know that all mercantile transactions of people, such as buying and selling, exchange and hire, comprehend always two notions and four numbers, which are stated by the enquirer; namely, measure and price, and quantity and sum. The number which expresses the measure is inversely proportionate to the number which expresses the sum, and the number of the price inversely proportionate to that of the quantity. Three of these four numbers are always known, one is unknown, and this is implied when the person inquiring says how much? and it is the object of the question. The computation in such instances is this, that you try the three given numbers; two of them must necessarily be inversely proportionate the one to the other. Then you multiply these two proportionate numbers by each other, and you divide the product by the third given number, the proportionate of which is unknown. The quotient of this division is the unknown number, which the inquirer asked for; and it is inversely proportionate to the divisor.*

Examples.—For the first case: If you are told, "ten (49) for six, how much for four?" then ten is the measure;

* If $a$ is given for $b$, and $A$ for $B$, then $a : b :: A : B$ or $aB = bA \therefore a = \frac{bA}{B}$ and $b = \frac{aB}{A}$. 
six is the price; the expression *how much* implies the unknown number of the quantity; and *four* is the number of the sum. The number of the measure, which is *ten*, is inversely proportionate to the number of the sum, namely, *four*. Multiply, therefore, ten by four, that is to say, the two known proportionate numbers by each other; the product is forty. Divide this by the other known number, which is that of the price, namely, six. The quotient is six and two-thirds; it is the unknown number, implied in the words of the question “*how much*?” it is the quantity, and inversely proportionate to the six, which is the price.

*For the second case:* Suppose that some one ask this question: “ten for eight, what must be the sum for four?” This is also sometimes expressed thus: “What must be the price of four of them?” Ten is the number of the measure, and is inversely proportionate to the unknown number of the sum, which is involved in the expression *how much* of the statement. Eight is the number of the price, and this is inversely proportionate to the known number of the quantity, namely, *four*. Multiply now the two known proportionate numbers one by the other, that is to say, four by eight. The product is thirty-two. Divide this by the other known number, which is that of the measure, namely, ten. The quotient is three and one-fifth; this is the number of the sum, and inversely proportionate to the ten which was the divisor. In this manner all computations in matters of business may be solved.
If somebody says, "a workman receives a pay of ten dirhems per month; how much must be his pay for six days?" Then you know that six days are one-fifth of the month; and that his portion of the dirhems must be proportionate to the portion of the month. You calculate it by observing that one month, or thirty days, is the measure, ten dirhems the price, six days the quantity, and his portion the sum. Multiply the price, that is, ten, by the quantity, which is proportionate to it, namely, six; the product is sixty. Divide this by thirty, which is the known number of the measure. The quotient is two dirhems, and this is the sum.

This is the proceeding by which all transactions concerning exchange or measures or weights are settled.

MENSURATION.

Know that the meaning of the expression "one by one" is mensuration: one yard (in length) by one yard (in breadth) being understood.

Every quadrangle of equal sides and angles, which has one yard for every side, has also one for its area. Has such a quadrangle two yards for its side, then the area of the quadrangle is four times the area of a quadrangle, the side of which is one yard. The same takes place with three by three, and so on, ascending or descending: for instance, a half by a half, which gives
a quarter, or other fractions, always following the same rule. A quadrate, every side of which is half a yard, is equal to one-fourth of the figure which has one yard for its side. In the same manner, one-third by one-third, or one-fourth by one-fourth, or one-fifth by one-fifth, or two-thirds by a half, or more or less than this, always according to the same rule.

One side of an equilateral quadrangular figure, taken once, is its root; or if the same be multiplied by two, then it is like two of its roots, whether it be small or great.

If you multiply the height of any equilateral triangle by the moiety of the basis upon which the line marking the height stands perpendicularly, the product gives the area of that triangle.

In every equilateral quadrangle, the product of one diameter multiplied by the moiety of the other will be equal to the area of it.

In any circle, the product of its diameter, multiplied by three and one-seventh, will be equal to the periphery. This is the rule generally followed in practical life, though it is not quite exact. The geometricians have two other methods. One of them is, that you multiply the diameter by itself; then by ten, and hereafter take the root of the product; the root will be the periphery. The other method is used by the astronomers among them: it is this, that you multiply the diameter by sixty-two thousand eight hundred and thirty-two and then divide the product by twenty
thousand; the quotient is the periphery. Both methods come very nearly to the same effect.*

If you divide the periphery by three and one-seventh, the quotient is the diameter.

The area of any circle will be found by multiplying the moiety of the circumference by the moiety of the diameter; since, in every polygon of equal sides and (52) angles, such as triangles, quadrangles, pentagons, and so on, the area is found by multiplying the moiety of the circumference by the moiety of the diameter of the middle circle that may be drawn through it.

If you multiply the diameter of any circle by itself, and subtract from the product one-seventh and half one-seventh of the same, then the remainder is equal to the area of the circle. This comes very nearly to the same result with the method given above. †

Every part of a circle may be compared to a bow. It must be either exactly equal to half the circumference, or less or greater than it. This may be ascertained by the arrow of the bow. When this becomes equal to the moiety of the chord, then the arc is

* The three formulas are,

1st, \(3\frac{1}{7}d = p\) i.e. \(3.1428d\)

2d, \(\sqrt{10d^2} = p\) i.e. \(3.16227d\)

3d, \(\frac{d \times 62832}{20000} = p\) i.e. \(3.14166d\)

† The area of a circle whose diameter is \(d\) is \(\frac{\pi d^2}{4} = \frac{22}{7} \times \frac{d^2}{4} = (1 - \frac{1}{7} - \frac{1}{2 \times 7})d^2\).
exactly the moiety of the circumference: is it shorter than the moiety of the chord, then the bow is less than half the circumference; is the arrow longer than half the chord, then the bow comprises more than half the circumference.

If you want to ascertain the circle to which it belongs, multiply the moiety of the chord by itself, divide it by the arrow, and add the quotient to the arrow, the sum is the diameter of the circle to which this bow belongs.

If you want to compute the area of the bow, multiply the moiety of the diameter of the circle by the moiety of the bow, and keep the product in mind. Then subtract the arrow of the bow from the moiety of the diameter of the circle, if the bow is smaller than half the circle; or if it is greater than half the circle, subtract half the diameter of the circle from the arrow of the bow. Multiply the remainder by the moiety of the chord of the bow, and subtract the product from that which you have kept in mind if the bow is smaller than the moiety of the circle, or add it thereto if the bow is greater than half the circle. The sum after the addition, or the remainder after the subtraction, is the area of the bow.

The bulk of a quadrangular body will be found by multiplying the length by the breadth, and then by the height.

If it is of another shape than the quadrangular (for instance, circular or triangular), so, however, that a
line representing its height may stand perpendicularly on its basis, and yet be parallel to the sides, you must calculate it by ascertaining at first the area of its basis. This, multiplied by the height, gives the bulk of the body.

Cones and pyramids, such as triangular or quadrangular ones, are computed by multiplying one-third of the area of the basis by the height.

Observe, that in every rectangular triangle the two short sides, each multiplied by itself and the products added together, equal the product of the long side multiplied by itself.

The proof of this is the following. We draw a quadrangle, with equal sides and angles \( A B C D \). We divide the line \( AC \) into two moieties in the point \( H \), from which we draw a parallel to the point \( R \). Then we divide, also, the line \( AB \) into two moieties at the point \( T \), and draw a parallel to the point \( G \). Then the quadrate \( A B C D \) is divided into four quadrangles of equal sides and angles, and of equal area; namely, the squares \( AK, CK, BK, \) and \( DK \). Now, we draw from the point \( H \) to the point \( T \) a line which divides the quadrangle \( AK \) into two equal parts: thus there arise two triangles from the quadrangle, namely, the triangles \( ATH \) and \( HKT \). We know that \( AT \) is the moiety of \( AB \), and that \( AH \) is equal to it, being the moiety of \( AC \); and the line \( TH \) joins them opposite the right angle. In the same manner we draw lines from \( T \) to \( R \), and from \( R \) to \( G \), and from \( G \) to \( H \). Thus from
all the squares eight equal triangles arise, four of which must, consequently, be equal to the moiety of the great quadrate $AD$. We know that the line $AT$ multiplied by itself is like the area of two triangles, and $AK$ gives the area of two triangles equal to them; the sum of them is therefore four triangles. But the line $HT$, multiplied by itself, gives likewise the area of four such triangles. We perceive, therefore, that the sum of $AT$ multiplied by itself, added to $AH$ multiplied by itself, is equal to $TH$ multiplied by itself. This is the observation which we were desirous to elucidate. Here is the figure to it:

Quadrangles are of five kinds: firstly, with right angles and equal sides; secondly, with right angles and unequal sides; thirdly, the rhombus, with equal sides and unequal angles; fourthly, the rhomboid, the length of which differs from its breadth, and the angles of which are unequal, only that the two long and the two short sides are respectively of equal length; fifthly, quadrangles with unequal sides and angles.

First kind.—The area of any quadrangle with equal sides and right angles, or with unequal sides and right
angles, may be found by multiplying the length by the breadth. The product is the area. For instance: a quadrangular piece of ground, every side of which has five yards, has an area of five-and-twenty square yards. Here is its figure.

Second kind.—A quadrangular piece of ground, the two long sides of which are of eight yards each, while the breadth is six. You find the area by multiplying six by eight, which yields forty-eight yards. Here is (56) the figure to it:

Third kind, the Rhombus.—Its sides are equal: let each of them be five, and let its diagonals be, the one eight and the other six yards. You may then compute the area, either from one of the diagonals, or from both. As you know them both, you multiply the one by the moiety of the other, the product is the area: that is to say, you multiply eight by three, or six by four; this yields twenty-four yards, which is the area.
If you know only one of the diagonals, then you are aware, that there are two triangles, two sides of each of which have every one five yards, while the third is the diagonal. Hereafter you can make the computation according to the rules for the triangles.* This is the figure:

![Diagram of a rhombus]

* If the two diagonals are \( d \) and \( d' \), and the side \( s \), the area of the rhombus is

\[
\frac{dd'}{2} = d \times \sqrt{s^2 - \frac{d'^2}{4}}.
\]
this: if you multiply every one of its two short sides by itself, and add the products, their sum is more than the long side alone multiplied by itself. The definition of the obtuse-angled triangle is this: if you multiply its two short sides each by itself, and then add the products, their sum is less than the product of the long side multiplied by itself.

The rectangular triangle has two cathetes and an hypotenuse. It may be considered as the moiety of a quadrangle. You find its area by multiplying one of its cathetes by the moiety of the other. The product is the area.

Examples.—A rectangular triangle; one cathete being six yards, the other eight, and the hypotenuse ten. You make the computation by multiplying six by four: this gives twenty-four, which is the area. Or if you prefer, you may also calculate it by the height, which rises perpendicularly from the longest side of it: for the two short sides may themselves be considered as two heights. If you prefer this, you multiply the height by the moiety of the basis. The product is the area. This is the figure:

Second kind.—An equilateral triangle with acute angles, every side of which is ten yards long. Its area
may be ascertained by the line representing its height and the point from which it rises.* Observe, that in every isosceles triangle, a line to represent the height drawn to the basis rises from the latter in a right angle, and the point from which it proceeds is always situated in the midst of the basis; if, on the contrary, the two sides are not equal, then this point never lies in the middle of the basis. In the case now before us we perceive, that towards whatever side we may draw the line which is to represent the height, it must necessarily always fall in the middle of it, where the length of the basis is five. Now the height will be ascertained thus. You multiply five by itself; then multiply one of the sides, that is ten, by itself, which gives a hundred. Now you subtract from this the product of five multiplied by itself, which is twenty-five. (59) The remainder is seventy-five, the root of which is the height. This is a line common to two rectangular triangles. If you want to find the area, multiply the root of seventy-five by the moiety of the basis, which is five. This you perform by multiplying at first five by itself; then you may say, that the root of seventy-five is to be multiplied by the root of twenty-five. Multiply seventy-five by twenty-five. The product is one thousand eight hundred and seventy-five; take its root, it is

* The height of the equilateral triangle whose side is 10, is $\sqrt{10^2 - 5^2} = \sqrt{75}$, and the area of the triangle is $5\sqrt{75} = 25\sqrt{3}$
the area: it is forty-three and a little. * Here is the figure:

There are also acute-angled triangles, with different sides. Their area will be found by means of the line representing the height and the point from which it proceeds. Take, for instance, a triangle, one side of which is fifteen yards, another fourteen, and the third thirteen yards. In order to find the point from which the line marking the height does arise, you may take for the basis any side you choose; e.g. that which is fourteen yards long. The point from which the line representing the height does arise, lies in this basis at an unknown distance from either of the two other sides. Let us try to find its unknown distance from the side which is thirteen yards long. Multiply this distance by itself; it becomes an [unknown] square. Subtract this from thirteen multiplied by itself; that is, one hundred and sixty-nine. The remainder is one hundred and sixty-nine less a square. The root from this is the height. The remainder of the basis is fourteen less thing. We multiply this by itself; it becomes one hundred and ninety-six, and a square less twenty-

* The root is 43.3 +
eight things. We subtract this from fifteen multiplied by itself; the remainder is twenty-nine dirhems and twenty-eight things less one square. The root of this is the height. As, therefore, the root of this is the height, and the root of one hundred and sixty-nine less square is the height likewise, we know that they both are the same.* Reduce them, by removing square against square, since both are negatives. There remain twenty-nine [dirhems] plus twenty-eight things, which are equal to one hundred and sixty-nine. Subtract now twenty-nine from one hundred and sixty-nine. The remainder is one hundred and forty, equal to twenty-eight things. One thing is, consequently, five. This is the distance of the said point from the side of thirteen yards. The complement of the basis towards the other side is nine. Now in order to find the height, you multiply five by itself, and subtract it from the contiguous side, which is thirteen, multiplied by itself. The remainder is one hundred and forty-four. Its root is the height. It is twelve. The height forms always two right angles with the basis, and it is called the column, on account of its standing perpendicularly. Multiply the height into half the basis, which is seven. The

\[ \sqrt{169 - x^2} = 29 + 28x - x^2 \]

\[ 163 = 29 + 28x \]

\[ 140 = 28x \]

\[ 5 = x \]
product is eighty-four, which is the area. Here is the figure:

\[ \text{The third species is that of the obtuse-angled triangle with one obtuse angle and sides of different length. For instance, one side being six, another five, and the third nine. The area of such a triangle will be found by means of the height and of the point from which a line representing the same arises. This point can, within such a triangle, lie only in its longest side. Take therefore this as the basis: for if you choose to take one of the short sides as the basis, then this point would fall beyond the triangle. You may find the distance of this point, and the height, in the same manner, which I have shown in the acute-angled triangle; the whole computation is the same. Here is the figure:} \]

\[ \text{We have above treated at length of the circles, of their qualities and their computation. The following (62) is an example: If a circle has seven for its diameter, then it has twenty-two for its circumference. Its area you find in the following manner: Multiply the moiety} \]
of the diameter, which is three and a half, by the moiety of the circumference, which is eleven. The product is thirty-eight and a half, which is the area. Or you may also multiply the diameter, which is seven, by itself: this is forty-nine; subtracting herefrom one-seventh and half one-seventh, which is ten and a half, there remain thirty-eight and a half, which is the area. Here is the figure:

If some one inquires about the bulk of a pyramidal pillar, its base being four yards by four yards, its height ten yards, and the dimensions at its upper extremity two yards by two yards; then we know already that every pyramid is decreasing towards its top, and that one-third of the area of its basis, multiplied by the height, gives its bulk. The present pyramid has no top. We must therefore seek to ascertain what is wanting in its height to complete the top. We observe, that the proportion of the entire height to the ten, which we have now before us, is equal to the proportion of four to two. Now as two is the moiety of four, ten must likewise be the moiety of the entire height, and the whole height of the pillar must be twenty yards. At present we take one-third of the area of the basis, that is, five and one-third, and multiply it by the length, which is twenty. The product is one hundred (63)
and six yards and two-thirds. Herefrom we must then subtract the piece, which we have added in order to complete the pyramid. This we perform by multiplying one and one-third, which is one-third of the product of two by two, by ten: this gives thirteen and a third. This is the piece which we have added in order to complete the pyramid. Subtracting this from one hundred and six yards and two-thirds, there remain ninety-three yards and one-third: and this is the bulk of the mutilated pyramid. This is the figure:

![Diagram of a pyramid](image)

If the pillar has a circular basis, subtract one-seventh and half a seventh from the product of the diameter multiplied by itself, the remainder is the basis.

If some one says: "There is a triangular piece of land, two of its sides having ten yards each, and the basis twelve; what must be the length of one side of a quadrate situated within such a triangle?" the solution is this. At first you ascertain the height of the triangle, by multiplying the moiety of the basis, (which is six) by itself, and subtracting the product, which is thirty-six, from one of the two short sides multiplied by itself, which is one hundred; the remainder is
sixty-four: take the root from this; it is eight. This (64) is the height of the triangle. Its area is, therefore, forty-eight yards: such being the product of the height multiplied by the moiety of the basis, which is six. Now we assume that one side of the quadrate inquired for is thing. We multiply it by itself; thus it becomes a square, which we keep in mind. We know that there must remain two triangles on the two sides of the quadrate, and one above it. The two triangles on both sides of it are equal to each other: both having the same height and being rectangular. You find their area by multiplying thing by six less half a thing, which gives six things less half a square. This is the area of both the triangles on the two sides of the quadrate together. The area of the upper triangle will be found by multiplying eight less thing, which is the height, by half one thing. The product is four things less half a square. This altogether is equal to the area of the quadrate plus that of the three triangles: or, ten things are equal to forty-eight, which is the area of the great triangle. One thing from this is four yards and four-fifths of a yard; and this is the length of any side of the quadrate. Here is the figure:
ON LEGACIES.

On Capital, and Money lent.

(65) "A man dies, leaving two sons behind him, and bequeathing one-third of his capital to a stranger. He leaves ten dirhems of property and a claim of ten dirhems upon one of the sons."

Computation: You call the sum which is taken out of the debt thing. Add this to the capital, which is ten dirhems. The sum is ten and thing. Subtract one-third of this, since he has bequeathed one-third of his property, that is, three dirhems and one-third of thing. The remainder is six dirhems and two-thirds of thing. Divide this between the two sons. The portion of each of them is three dirhems and one-third plus one-third of thing. This is equal to the thing which was sought for.* Reduce it, by removing one-third from

* If a father dies, leaving $n$ sons, one of whom owes the father a sum exceeding an $n$th part of the residue of the father's estate, after paying legacies, then such son retains the whole sum which he owes the father; part, as a set-off against his share of the residue, the surplus as a gift from the father.

In the present example, let each son's share of the residue be equal to $x$.

\[
\frac{2}{3} \left[ 10 + x \right] = 2x \\
1 + x - 3x = 10 = 2x \\
x = 5.
\]

The stranger receives 5; and the son, who is not indebted to the father, receives 5.
thing, on account of the other third of thing. There remain two-thirds of thing, equal to three dirhems and one-third. It is then only required that you complete the thing, by adding to it as much as one half of the same; accordingly, you add to three and one-third as much as one-half of them: This gives five dirhems, which is the thing that is taken out of the debts.

If he leaves two sons and ten dirhems of capital and a demand of ten dirhems against one of the sons, and bequeaths one-fifth of his property and one dirhem to a stranger, the computation is this: Call the sum which is taken out of the debt, thing. Add this to the property; the sum is thing and ten dirhems. Subtract one-fifth of this, since he has bequeathed one-fifth of (66) his capital, that is, two dirhems and one-fifth of thing; the remainder is eight dirhems and four-fifths of thing. Subtract also the one dirhem which he has bequeathed; there remain seven dirhems and four-fifths of thing. Divide this between the two sons; there will be for each of them three dirhems and a half plus two-fifths of thing; and this is equal to one thing.* Reduce it by subtracting two-fifths of thing from thing. Then you have three-fifths of thing, equal to three dirhems and a half. Complete the thing by adding to it two-thirds of the same: add as much to the three dirhems and a half,

\[ \frac{3}{5} [10 + x] - 1 = 2x \quad \therefore \quad \frac{3}{5} [10 + x] - \frac{1}{2} = x \]

\[ \therefore \quad 3\frac{1}{2} = \frac{3}{2} x \quad \therefore \quad x = \frac{3}{5} = 5\frac{5}{6} \]

The stranger receives \( \frac{1}{3} [10 + \frac{3}{5}] + 1 = 4\frac{1}{6} \)
namely, two dirhems and one-third; the sum is five and five-sixths. This is the thing, or the amount which is taken from the debt.

If he leaves three sons, and bequeaths one-fifth of his property less one dirhem, leaving ten dirhems of capital and a demand of ten dirhems against one of the sons, the computation is this: You call the sum which is taken from the debt thing. Add this to the capital; it gives ten and thing. Subtract from this one-fifth of it for the legacy: it is two dirhems and one-fifth of thing. There remain eight dirhems and four-fifths of thing; add to this one dirhem, since he stated "less one dirhem." Thus you have nine dirhems and four-fifths of thing. Divide this between the three sons. There will be for each son three dirhems, and one-fifth and one-third and one-fifth of thing. This equals one thing.\(^*\) Subtract one-fifth and one-third of one-fifth of thing from thing. There remain eleven-fifteenths of thing, equal to three dirhems. It is now required to complete the thing. For this purpose, add to it four-elevenths, and do the same with the three dirhems, by adding to them one dirhem and one-eleventh. Then you have four dirhems and one-eleventh, which are equal to thing. This is the sum which is taken out of the debt.

\(^*\) \(\frac{1}{3}[10+x]+1=3x\) \(\therefore 9=2\frac{1}{3}x\) \(\therefore \frac{4}{11}\) or \(4\frac{1}{11}=x\)

The stranger receives \(\frac{1}{3}[10+4\frac{1}{11}]-1=1\frac{9}{11}\)
On another Species of Legacy.

"A man dies, leaving his mother, his wife, and two brothers and two sisters by the same father and mother with himself; and he bequeaths to a stranger one-ninth of his capital."

Computation:* You constitute their shares by taking them out of forty-eight parts. You know that if you take one-ninth from any capital, eight-ninths of it will remain. Add now to the eight-ninths one-eighth of the same, and to the forty-eight also one-eighth of them, namely, six, in order to complete your capital. This gives fifty-four. The person to whom one-ninth is bequeathed receives six out of this, being one-ninth of the whole capital. The remaining forty-eight will be distributed among the heirs, proportionally to their legal shares.

If the instance be: "A woman dies, leaving her husband, a son, and three daughters, and bequeathing

* It appears in the sequel (p. 96) that a widow is entitled to \(\frac{1}{6}\)th, and a mother to \(\frac{1}{6}\)th of the residue; \(\frac{1}{6} + \frac{1}{6} = \frac{1}{3}\), leaving \(\frac{3}{6}\) of the residue to be distributed between two brothers and two sisters; that is, \(\frac{1}{4}\) between a brother and a sister; but in what proportion these 17 parts are to be divided between the brother and sister does not appear in the course of this treatise.

Let the whole capital of the testator = 1

and let each 48th share of the residue = x

\[\frac{8}{9} = 48x \quad \therefore \frac{1}{6} = 6x \quad \therefore \frac{1}{3} = x\]

that is, each 48th part of the residue = \(\frac{1}{3}\)th of the whole capital.
to a stranger one-eighth and one-seventh of her capital;" then you constitute the shares of the heirs, by taking them out of twenty.* Take a capital, and subtract from it one-eighth and one-seventh of the same. The remainder is, a capital less one-eighth and one-seventh. Complete your capital by adding to that which you have already, fifteen forty-one parts. Multiply the parts of the capital, which are twenty, by forty-one; the product is eight hundred and twenty. Add to it fifteen forty-one parts of the same, which are three hundred: the sum is one thousand one hundred and twenty parts. The person to whom one-eighth and one-seventh were bequeathed, receives one-eighth and one-seventh of this. One seventh of it is one hundred and sixty, and one-eighth one hundred and forty. Subtracting this, there remain eight hundred and twenty parts for the heirs, proportionally to their legal shares.

* A husband is entitled to \( \frac{1}{4} \)th of the residue, and the sons and daughters divide the remaining \( \frac{3}{4} \)ths of the residue in such proportion, that a son receives twice as much as a daughter. In the present instance, as there are three daughters and one son, each daughter receives \( \frac{1}{2} \) of \( \frac{3}{4} \), \( \frac{3}{20} \), of the residue, and the son, \( \frac{6}{20} \). Since the stranger takes \( \frac{1}{8} + \frac{1}{7} = \frac{15}{56} \) of the capital, the residue = \( \frac{41}{56} \) of the capital, and each \( \frac{3}{20} \)th share of the residue = \( \frac{1}{20} \times \frac{41}{56} = \frac{41}{1120} \) of the capital. The stranger, therefore, receives \( \frac{15}{56} = \frac{15}{56} \times \frac{20}{20} = \frac{300}{1120} \) of the capital.
On another Species of Legacies,* viz.

If nothing has been imposed on some of the heirs,† and something has been imposed on others; the legacy amounting to more than one-third. It must be known, that the law for such a case is, that if more than one-third of the legacy has been imposed on one of the heirs, this enters into his share; but that also those on whom nothing has been imposed must, nevertheless, contribute one-third.

Example: "A woman dies, leaving her husband, a son, and her mother. She bequeaths to a person two-fifths, and to another one-fourth of her capital. She imposes the two legacies together on her son, and on her mother one moiety (of the mother's share of the residue); on her husband she imposes nothing but one-third, (which he must contribute, according to the

* The problems in this chapter may be considered as belonging rather to Law than to Algebra, as they contain little more than enunciations of the law of inheritance in certain complicated cases.

† If some heirs are, by a testator, charged with payment of bequests, and other heirs are not charged with payment of any bequests whatever: if one bequest exceeds in amount \( \frac{1}{3} \)d of the testator's whole property; and if one of his heirs is charged with payment of more than \( \frac{1}{3} \)d of such bequest; then, whatever share of the residue such heir is entitled to receive, the like share must he pay of the bequest where- with he is charged, and those heirs whom the testator has not charged with any payment, must each contribute towards paying the bequests a third part of their several shares of the residue.
law)."* Computation: You constitute the shares of the
(69) heritage, by taking them out of twelve parts: the son
receives seven of them, the husband three, and the
mother two parts. You know that the husband must
give up one-third of his share; accordingly he retains
twice as much as that which is detracted from his share
for the legacy. As he has three parts in hand, one of
these falls to the legacy, and the remaining two parts
he retains for himself. The two legacies together are
imposed upon the son. It is therefore necessary to
subtract from his share two-fifths and one-fourth of the
same. He thus retains seven twentieths of his entire
original share, dividing the whole of it into twenty
equal parts. The mother retains as much as she con-
tributes to the legacy; this is one (twelfth part), the
entire amount of what she had received being two parts.

* If the bequests stated in the present example were charged
on the heirs collectively, the husband would be entitled to \( \frac{1}{4} \),
the mother to \( \frac{1}{6} \) of the residue: \( \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \); the remainder \( \frac{7}{12} \)
would be the son's share of the residue; but since the
bequests, \( \frac{2}{5} + \frac{1}{4} = \frac{13}{20} \) of the capital, are charged upon the son
and mother, the law throws a portion of the charge on the
husband.

The Husband contributes \( \frac{1}{4} \times \frac{1}{3} = 20 \times \frac{1}{240} \), and retains \( \frac{1}{4} \times \frac{2}{3} = 40 \times \frac{1}{240} \)
The Mother \( \frac{1}{6} \times \frac{1}{3} = 20 \times \frac{1}{240} \), \( \frac{1}{6} \times \frac{1}{2} = 20 \times \frac{1}{240} \)
The Son \( \frac{7}{12} \times \frac{1}{2} = 91 \times \frac{1}{240} \), \( \frac{7}{20} = 49 \times \frac{1}{240} \)

Total contributed = \( \frac{131}{240} \) Total retained = \( \frac{109}{240} \)

\( \frac{2}{5} + \frac{1}{4} = 20 + \frac{5}{20} = \frac{13}{20} \)
The Legatee, to whom the \( \frac{2}{5} \) are bequeathed, receives \( \frac{8}{13} \times \frac{131}{240} = \frac{8 \times 131}{3120} \)
The Legatee, to whom \( \frac{1}{4} \) is bequeathed, receives \( \frac{5}{13} \times \frac{131}{240} = \frac{5 \times 131}{3120} \).
Take now a sum, one-fourth of which may be divided into thirds, or of one-sixth of which the moiety may be taken; this being again divisible by twenty. Such a capital is two hundred and forty. The mother receives one-sixth of this, namely, forty; twenty from this fall to the legacy, and she retains twenty for herself. The husband receives one-fourth, namely, sixty; from which twenty belong to the legacy, so that he retains forty. The remaining hundred and forty belong to the son; the legacy from this is two-fifths and one-fourth, or ninety-one; so that there remain forty-nine. The entire sum for the legacies is, therefore, one hundred and thirty-one, which must be divided among the two legatees. The one to whom two-fifths were bequeathed, receives eight-thirteenths of this; the one to whom one-fourth was devised, receives five-thirteenths. If you wish distinctly to express the shares of the two legatees, you need only to multiply (70) the parts of the heritage by thirteen, and to take them out of a capital of three thousand one hundred and twenty.

But if she had imposed on her son (payment of) the two-fifths to the person to whom the two-fifths were bequeathed, and of nothing to the other legatee; and upon her mother (payment of) the one-fourth to the person to whom one-fourth was granted, and of nothing to the other legatee; and upon her husband nothing besides the one-third (which he must according to law contribute) to both; then you know that this one-third
comes to the advantage of the heirs collectively; and
the legatee of the two-fifths receives eight-thirteenths,
and the legatee of the one-fourth receives five-thir-
teenth from it. Constitute the shares as I have shown
above, by taking twelve parts; the husband receives
one-fourth of them, the mother one-sixth, and the son
that which remains.* Computation: You know that at
all events the husband must give up one-third of his
share, which consists of three parts. The mother must
likewise give up one-third, of which each legatee par-
takes according to the proportion of his legacy. Be-
sides, she must pay to the legatee to whom one-fourth is
bequeathed, and whose legacy has been imposed on her,
as much as the difference between the one-fourth and his

\[
\frac{8}{3} + \frac{1}{4} = \frac{8+5}{20} = \frac{13}{20}
\]

The Husband, who would be entitled to \(\frac{1}{4}\) of the residue, is
not charged by the Testator with any bequest.
The Mother who would be entitled to \(\frac{1}{6}\) of the residue, is
charged with the payment of \(\frac{1}{4}\) to the Legatee A.
The Son, who would be entitled to \(\frac{7}{12}\) of the residue, is
charged with payment of \(\frac{2}{3}\) to the Legatee B.

The Husband \(\frac{1}{4} \times \frac{3}{3} = 780 \times \frac{1}{9360} \); retains \(\frac{1}{4} \times \frac{2}{3} = \frac{1260}{9360}\)

The Mother \(\frac{1}{6}[\frac{1}{4} + \frac{8}{13} \times \frac{1}{3}] = 710 \times \frac{1}{9360} \); retains \(\frac{8500}{9360}\)

The Son \(\frac{7}{12}[\frac{2}{3} + \frac{5}{13} \times \frac{1}{3}] = 2884 \times \frac{1}{9360} \); retains \(\frac{2476}{9360}\)

Total contributed = \(\frac{4374}{9360}\); Total retained = \(\frac{4374}{9360}\)

The Legatee A, to whom \(\frac{1}{3}\) is
bequeathed, receives \(\frac{5}{13} \times \frac{4374}{9360} = \frac{5 \times 4374}{9360}\)

The Legatee B, to whom \(\frac{2}{3}\) are
bequeathed, receives \(\frac{8}{13} \times \frac{4374}{9360} = \frac{8 \times 4374}{9360}\)
portion of the one-third, namely, nineteen one hundred and fifty-sixths of her entire share, considering her share as consisting of one hundred and fifty-six parts. His portion of the one-third of her share is twenty parts. But what she gives him is one-fourth of her entire share, namely, thirty-nine parts. One third of her share is taken for both legacies, and besides nineteen parts which she must pay to him alone. The son gives to the legatee to whom two-fifths are bequeathed as much as the difference between two-fifths of his (the son’s) share and the legatee’s portion of the one-third, namely, thirty-eight one hundred and ninety-fifths of his (the son’s) entire share, besides the one-third of it which is taken off from both legacies. The portion which he (the legatee) receives from this one-third, is eight-thirteenths of it, namely, forty (one hundred and ninety-fifths); and what the son contributes of the two-fifths from his share is thirty-eight. These together make seventy-eight. Consequently, sixty-five will be taken from the son, as being one-third of his share, for both legacies, and besides this he gives thirty-eight to the one of them in particular. If you wish to express the parts of the heritage distinctly, you may do so with nine hundred and sixty-four thousand and eighty.

On another Species of Legacies.

“A man dies, leaving four sons and his wife; and bequeathing to a person as much as the share of one
of the sons less the amount of the share of the widow."
Divide the heritage into thirty-two parts. The widow
receives one-eighth, * namely, four; and each son seven.
Consequently the legatee must receive three-sevenths of
the share of a son. Add, therefore, to the heritage
three-sevenths of the share of a son, that is to say,
three parts, which is the amount of the legacy. This
gives thirty-five, from which the legatee receives three;
and the remaining thirty-two are distributed among
the heirs proportionably to their legal shares.

If he leaves two sons and a daughter, + and bequeathes
to some one as much as would be the share of a third
son, if he had one; then you must consider, what
would be the share of each son, in case he had three.
Assume this to be seven, and for the entire heritage

* A widow is entitled to \( \frac{1}{8} \)th of the residue; therefore
\( \frac{7}{8} \)ths of the residue are to be distributed among the sons of
the testator. Let \( x \) be the stranger's legacy. The widow's
share \( = \frac{1-x}{8} \); each son's share \( = \frac{1}{4} \times \frac{7}{8} \left[ 1-x \right] \); and a son's
share, minus the widow's share \( = \left[ \frac{1}{4} - 1 \right] \frac{1-x}{8} = \frac{3}{4} \times \frac{1-x}{8} \).

\[ x = \frac{3}{4} \cdot \frac{1-x}{8} \quad \Rightarrow \quad x = \frac{3}{35} \quad 1-x = \frac{32}{35} \quad \text{A son's share} = \frac{7}{35} \quad \text{the widow's share} = \frac{4}{35} .

† A son is entitled to receive twice as much as a daughter.
Were there three sons and one daughter, each son would receive
\( \frac{2}{3} \)ths of the residue. Let \( x \) be the stranger's legacy.

\[ \frac{2}{3} \left[ 1-x \right] = x \quad \Rightarrow \quad x = \frac{6}{7} \quad \text{and} \quad 1-x = \frac{7}{7} .
\]

Each Son's share \( = \frac{2}{3} \left[ 1-x \right] = \frac{2}{3} \times \frac{6}{7} = \frac{4}{7} \)
The Daughter's share \( = \frac{1}{3} \left[ 1-x \right] \quad \Rightarrow \quad \frac{7}{15} \)
The Stranger's legacy \( = \frac{2}{3} \quad \Rightarrow \quad \frac{14}{15} \)
take a number, one-fifth of which may be divided into sevenths, and one-seventh of which may be divided into fifths. Such a number is thirty-five. Add to it two-sevenths of the same, namely, ten. This gives forty-five. Herefrom the legatee receives ten, each son fourteen, and the daughter seven.

If he leaves a mother, three sons, and a daughter, and bequeaths to some one as much as the share of one of his sons less the amount of the share of a second daughter, in case he had one; then you distribute the heritage into such a number of parts as may be divided among the actual heirs, and also among the same, if a second daughter were added to them.* Such a number is three hundred and thirty-six. The share of the second daughter, if there were one, would be thirty-five, and that of a son eighty; their difference is forty-five, and this is the legacy. Add to it three hundred and thirty-six, the sum is three hundred and eighty-one, which is the number of parts of the entire heritage.

* Let $x$ be the stranger’s legacy; $1-x$ is the residue.
A widow’s share of the residue is $\frac{1}{6}$th: there remains $\frac{5}{6}(1-x)$, to be distributed among the children.
Since there are 3 sons, and 1 daughter, $\left\{ \frac{2}{7} \times \frac{5}{6}(1-x) \right\}$
Were there 3 sons and 2 daughters, a daughter’s share would be $\left\{ \frac{1}{8} \times \frac{5}{6}(1-x) \right\}$

The difference $= \frac{9}{28} \times \frac{5}{6}(1-x)$

$\therefore x = \frac{45}{36} \cdot (1-x)$
$\therefore x = \frac{45}{36}$

$1-x = \frac{33}{36}$; the widow’s share $= \frac{36}{36}$
the daughter’s share $= \frac{34}{36}$
If he leaves three sons, and bequeatheth to some one as much as the share of one of his sons, less the share of a daughter, supposing he had one, plus one-third of the remainder of the one-third; the computation will be this:* distribute the heritage into such a number of parts as may be divided among the actual heirs, and also among them if a daughter were added to them. Such a number is twenty-one. Were a daughter among the heirs, her share would be three, and that of a son seven. The testator has therefore bequeathed to the (73) legatee four-sevenths of the share of a son, and one-third of what remains from one-third. Take therefore one-third, and remove from it four-sevenths of the share of a son. There remains one-third of the capital less four-sevenths of the share of a son. Subtract now one-third of what remains of the one-third, that is to say, one-ninth of the capital less one-seventh and one-third of the seventh of the share of a son; the remainder

* Since there are 3 sons, each son's share of the residue $=\frac{1}{3}$. Were there 3 sons and a daughter, the daughter's share would be $\frac{1}{7}$.

Let $x$ be the stranger's legacy, and $v$ a son's share

Then $1 - x = 3v$

but $x = \frac{4}{3} v + \frac{1}{3} \left[ \frac{1}{3} - \frac{4}{3} v \right]$

and $1 - x = \frac{2}{3} + \frac{1}{3} - \frac{4}{3} v - \frac{1}{3} \left[ \frac{1}{3} - \frac{4}{3} v \right] = 3v$

$\therefore \frac{2}{3} + \frac{1}{3} = 3v$

$\therefore \frac{3}{3} = \frac{1}{3} v$

$\therefore v = \frac{3}{3} = \frac{1}{3} v$

$\therefore x = \frac{4}{3} v = \text{a son's share}$

$x = \frac{4}{3} v = \text{the stranger's legacy.}$
is two-ninths of the capital less two-sevenths and two-thirds of a seventh of the share of a son. Add this to the two-thirds of the capital; the sum is eight-ninths of the capital less two-sevenths and two thirds of a seventh of the share of a son, or eight twenty-one parts of that share, and this is equal to three shares. Reduce this, you have then eight-ninths of the capital, equal to three shares and eight twenty-one parts of a share. Complete the capital by adding to eight-ninths as much as one-eighth of the same, and add in the same proportion to the shares. Then you find the capital equal to three shares and forty-five fifty-sixth parts of a share. Calculating now each share equal to fifty-six, the whole capital is two hundred and thirteen, the first legacy thirty-two, the second thirteen, and of the remaining one hundred and sixty-eight each son takes fifty-six.

On another Species of Legacies.

"A woman dies, leaving her daughter, her mother, and her husband, and bequeaths to some one as much as the share of her mother, and to another as much as one-ninth of her entire capital."* Computation: You begin by dividing the heritage into thirteen parts, two

* In the former examples (p. 90) when a husband and a mother were among the heirs, a husband was found to be entitled to $\frac{1}{4} = \frac{3}{13}$, and a mother to $\frac{1}{6} = \frac{2}{13}$ of the residue. Here a husband is stated to be entitled to $\frac{3}{13}$, and a mother to $\frac{2}{13}$ of the residue.
of which the mother receives. Now you perceive that the
legacies amount to two parts plus one-ninth of the entire capital. Subtracting this, there remains eight-ninths of the capital less two parts, for distribution among the heirs. Complete the capital, by making the eight-ninths less two parts to be thirteen parts, and adding two parts to it, so that you have fifteen parts, equal to eight-ninths of capital; then add to this one-eighth of the same, and to the fifteen parts add likewise one-eighth of the same, namely, one part and seven-eighths; then you have sixteen parts and seven-eighths. The person to whom one-ninth is bequeathed, receives one-ninth of this, namely, one part and seven-eighths; the other, to whom as much as the share of the mother is bequeathed, receives two parts. The remaining thirteen parts are divided among the heirs, according to their legal shares. You best determine the respective shares by dividing the whole heritage into one hundred and thirty-five parts.

If she has bequeathed as much as the share of the husband and one-eighth and one-tenth of the capital,*

\[
\begin{align*}
&\text{Let } \frac{1}{13} \text{ of the residue } = v \\
&1 - \frac{1}{9} - 2v = 13v \therefore \frac{8}{9} = 15v \\
&\therefore v = \frac{8}{135} \text{ of the capital} \\
&\text{A mother's share } = \frac{16}{135} \\
&\ast \frac{1}{8} + \frac{1}{10} = \frac{9}{35} \\
&A \text{ husband's share of the residue is } \frac{3}{13} \\
\therefore 1 - \frac{9}{35} - 3v = 13v \therefore \frac{31}{40} = 16v \\
\therefore v = \frac{31}{40} \text{; a husband's share } = \frac{93}{40} \\
&\text{The stranger's legacy } = \frac{237}{640}
\end{align*}
\]
then you begin by dividing the heritage into thirteen parts. Add to this as much as the share of the husband, namely, three; thus you have sixteen. This is what remains of the capital after the deduction of one-eighth and one-tenth, that is to say, of nine-fortieths. The remainder of the capital, after the deduction of one-eighth and one-tenth, is thirty-one fortieths of the same, which must be equal to sixteen parts. Complete your capital by adding to it nine thirty-one parts of the same, and multiply sixteen by thirty-one, which gives four hundred and ninety-six; add to this nine thirty-one parts of the same, which is one hundred and forty-four. The sum is six hundred and forty. Subtract one-eighth and one-tenth from it, which is one hundred and forty-four, and as much as the share of the husband, which is ninety-three. There remains four hundred and three, of which the husband receives ninety-three, the mother sixty-two, and every daughter one hundred and twenty-four.

If the heirs are the same,* but that she bequeaths to a person as much as the share of the husband, less one-ninth and one-tenth of what remains of the capital,

\begin{align*}
* \frac{1}{9} + \frac{1}{10} &= \frac{19}{90} \\
1 - 3v + \frac{1}{10} \left[ 1 - 3v \right] &= 13v \\
\therefore \frac{19}{90} \left[ 1 - 3 \right] &= 13v \\
\therefore \frac{19}{90} &= \left[ 13 + \frac{10}{39} \right] v \\
\therefore v &= \frac{19}{1497} \\
The\ husband's\ share &= \frac{327}{1497} \\
The\ stranger's\ legacy &= \frac{80}{1497}
\end{align*}
after the subtraction of that share, the computation is this: Divide the heritage into thirteen parts. The legacy from the whole capital is three parts, after the subtraction of which there remains the capital less three parts. Now, one-ninth and one-tenth of the remaining capital must be added, namely, one-ninth and one-tenth of the whole capital less one-ninth and one-tenth of three parts, or less nineteen-thirtieths of a part; this yields the capital and one-ninth and one-tenth less three parts and nineteen-thirtieths of a part, equal to thirteen parts. Reduce this, by removing the three parts and nineteen-thirtieths from your capital, and adding them to the thirteen parts. Then you have the capital and one-ninth and one-tenth of the same, equal to sixteen parts and nineteen-thirtieths of a part. Reduce this to one capital, by subtracting from it nineteen one-hundred-and-ninths. There remains a capital, equal to thirteen parts and eighty one-hundred-and-ninths. Divide each part into one hundred and nine parts, by multiplying thirteen by one hundred and nine, and add eighty to it. This gives one thousand four hundred and ninety-seven parts. The share of the husband from it is three hundred and twenty-seven parts.

If some one leaves two sisters and a wife,* and bequeaths to another person as much as the share of a

* When the heirs are a wife, and 2 sisters, they each inherit $\frac{1}{3}$ of the residue.
sister less one-eighth of what remains of the capital after the deduction of the legacy, the computation is this: You consider the heritage as consisting of twelve parts. Each sister receives one-third of what remains of the capital after the subtraction of the legacy; that is, of the capital less the legacy. You perceive that one-eighth of the remainder plus the legacy equals the share of a sister; and also, one-eighth of the remainder is as much as one-eighth of the whole capital less one-eighth of the legacy; and again, one-eighth of the capital less one-eighth of the legacy added to the legacy equals the share of a sister, namely, one-eighth of the capital and seven-eighths of the legacy. The whole capital is therefore equal to three-eighths of the capital plus three and five-eighth times the legacy. Subtract now from the capital three-eighths of the same. There remain five-eighths of the capital, equal to three and five-eighth times the legacy; and the entire capital is equal to five and four-fifth times the legacy. Consequently, if you assume the capital to be twenty-nine, the legacy is five, and each sister's share eight.

Let \( x \) be the stranger's legacy.

\[
\frac{1}{3} [1-x] = \text{a sister's share} \\
\frac{1}{3} [1-x] - \frac{1}{3} [1-x] = x \\
\therefore \frac{1}{2} [1-x] = x \\
\therefore \frac{5}{8} x = \frac{5}{8} x \\
\therefore x = \frac{5}{9} \\
\therefore 1-x = \frac{24}{9} \\
\text{and a sister's share} = \frac{8}{9}
\]
On another Species of Legacies.

"A man dies, and leaves four sons, and bequeaths to some person as much as the share of one of his sons; and to another, one-fourth of what remains after the deduction of the above share from one-third." You perceive that this legacy belongs to the class of those which are taken from one-third of the capital.* Computation: Take one-third of the capital, and subtract from it the share of a son. The remainder is one-third of the capital less the share. Then subtract from it one-fourth of what remains of the one-third, namely, one-fourth of one-third less one-fourth of the share. The remainder is one-fourth of the capital less three-fourths of the share. Add hereto two-thirds of the capital: then you have eleven-twelfths of the capital less three-fourths of a share, equal to four shares. Reduce this by removing the three-fourths of the share from the capital, and adding them to the four shares. Then you have eleven-twelfths of the capital, equal to four shares and three-fourths. Complete your capital, by adding to the four shares and three-fourths one-fourth of the same. Then you have five shares and two-elevenths,

---

* Let the first bequest = v; and the second = y

Then $1 - v - y = 4v$

i.e. $\frac{2}{3} + \frac{1}{3} - v - \frac{1}{4} \left[\frac{2}{3} - v\right] = 4v$

$\therefore \frac{2}{3} + \frac{3}{4} \left[\frac{2}{3} - v\right] = 4v$

$\therefore \frac{2}{3} + \frac{3}{4} = \left[4 + \frac{3}{4}\right] v \quad \therefore \quad \frac{11}{4} = \frac{9}{4} v$

$\therefore v = \frac{3}{11} \text{; the 2d bequest } = \frac{3}{7}$
equal to the capital. Suppose, now, every share to be eleven; then the whole square will be fifty-seven; one-third of this is nineteen; from this one share, namely, eleven, must be subtracted; there remain eight. The legatee, to whom one-fourth of this remainder was bequeathed, receives two. The remaining six are returned to the other two-thirds, which are thirty-eight. Their sum is forty-four, which is to be divided amongst the four sons; so that each son receives eleven.

If he leaves four sons, and bequeaths to a person as much as the share of a son, less one-fifth of what remains from one-third after the deduction of that share, then this is likewise a legacy, which is taken from one-third.* Take one-third, and subtract from it one share; there remains one-third less the share. Then return to it that which was excepted, namely, one-fifth of the one-third less one-fifth of the share. This gives one-third and one-fifth of one-third (or two-fifths) less one share and one-fifth of a share. Add this to two-thirds of the capital. The sum is, the capital and one-third of one-fifth of the capital less one share and one-fifth of a share, equal to four shares. Reduce this by removing one share and one-fifth from the capital,

\[ 1 - v + \frac{1}{5} \left[ \frac{2}{3} - v \right] - 4v \]

or \[ \frac{2}{3} \times \frac{1}{3} - v + \frac{1}{5} \left[ \frac{2}{3} - v \right] = 4v \]

or \[ \frac{2}{3} + \frac{6}{5} \left[ \frac{1}{3} - v \right] = 4v \]

\[ \therefore \frac{2}{3} + \frac{6}{5} = \left[ 4 + \frac{6}{5} \right] v \]

\[ \therefore \frac{15}{5} = \frac{26}{5} v \]

\[ \therefore v = \frac{8}{39} \text{, and the stranger's legacy} = \frac{7}{39} \]
and add to it the four shares. Then you have the capital and one-third of one-fifth of the capital, which are equal to five shares and one-fifth. Reduce this to one capital, by subtracting from what you have the moiety of one-eighth of it, that is to say, one-sixteenth. Then you find the capital equal to four shares and seven-eighths of a share. Assume now thirty-nine as capital; one-third of it will be thirteen, and one share eight; what remains of one-third, after the deduction of that share, is five, and one-fifth of this is one. Subtract now the one, which was excepted from the legacy; the remaining legacy then is seven; subtracting this from the one-third of the capital, there remain six. Add this to the two-thirds of the capital, namely, to the twenty-six parts, the sum is thirty-two; which, when distributed among the four sons, yields eight for each of them.

If he leaves three sons and a daughter,* and bequeathes to some person as much as the share of a

* Since there are three sons and one daughter, the daughter receives 1/4, and each son 3/4ths of the residue.

If the 1st legacy = v, the 2d = y, and therefore a daughter's share = v,

\[
1 - v - y = 7v; \quad \frac{1}{3} + \frac{1}{8} = \frac{11}{24}
\]

\[
\therefore \frac{5}{7} + \frac{2}{3} - v - \frac{1}{3} [\frac{2}{7} - v] = 7v
\]

i.e. \[
\frac{5}{7} + \frac{2}{3} [\frac{2}{7} - v] = 7v
\]

\[
\therefore \frac{5}{7} + \frac{19}{18} \times \frac{7}{1} = [7 + \frac{19}{2}]v
\]

\[
\therefore \frac{9}{7} = \frac{229}{2} v \quad \therefore = \frac{188}{18}
\]

The 2d legacy = \( y = \frac{9}{8} \)
daughter, and to another one-fifth and one-sixth of what remains of two-sevenths of the capital after the deduction of the first legacy; then this legacy is to be taken out of two-sevenths of the capital. Subtract from two-sevenths the share of the daughter: there remain two-sevenths of the capital less that share. Deduct from this the second legacy, which comprises (79) one-fifth and one-sixth of this remainder: there remain one-seventh and four-fifteenths of one-seventh of the capital less nineteen-thirtieths of the share. Add to this the other five-sevenths of the capital: then you have six-sevenths and four-fifteenths of one-seventh of the capital less nineteen thirtieths of the share, equal to seven shares. Reduce this, by removing the nineteen thirtieths, and adding them to the seven shares: then you have six-sevenths and four-fifteenths of one-seventh of the capital, equal to seven shares and nineteen-thirtieths. Complete your capital by adding to every thing that you have eleven ninety-fourths of the same; thus the capital will be equal to eight shares and ninety-nine one hundred and eighty-eighths. Assume now the capital to be one thousand six hundred and three; then the share of the daughter is one hundred and eighty-eight. Take two-sevenths of the capital; that is, four hundred and fifty-eight. Subtract from this the share, which is one hundred and eighty-eight; there remain two hundred and seventy. Remove one-fifth and one-sixth of this, namely, ninety-nine; the remainder is one hundred and seventy-one. Add thereto five-
sevenths of the capital, which is one thousand one hundred and forty-five. The sum is one thousand three hundred and sixteen parts. This may be divided into seven shares, each of one hundred and eighty-eight parts; then this is the share of the daughter, whilst every son receives twice as much.

If the heirs are the same, and he bequeaths to some person as much as the share of the daughter, and to another person one-fourth and one-fifth out of what remains from two-fifths of his capital after the deduction of the share; this is the computation:* You must observe that the legacy is determined by the two-fifths. Take two-fifths of the capital and subtract the shares: the remainder is, two-fifths of the capital less the share. Subtract from this remainder one-fourth and one-fifth of the same, namely, nine-twentieths of two-fifths, less as much of the share. The remainder is one-fifth and one-tenth of one-fifth of the capital less eleven-twentieths of the share. Add thereto three-fifths of the

\[
\begin{align*}
\frac{1}{4} + \frac{1}{5} &= \frac{9}{20} \\
\text{Let the 1st legacy} &= v = \text{a daughter's share} \\
\text{Let the 2d legacy} &= y \\
1 - v - y &= 7v \\
\therefore \frac{3}{5} + \frac{2}{5} - v - \frac{9}{20} \left[ \frac{2}{5} - v \right] &= 7v \\
\therefore \frac{3}{5} + \frac{11}{10} \left[ \frac{2}{5} - v \right] &= 7v \\
\therefore \frac{3}{5} + \frac{11}{10} \times \frac{1}{5} &= [7 + \frac{11}{20}] v \\
\therefore \frac{41}{5} &= \frac{151}{2} v \quad \therefore v = \frac{90}{155} \\
\text{and the 2d legacy, } y, &= \frac{90}{155}
\end{align*}
\]
capital: the sum is four-fifths and one-tenth of one-fifth of the capital, less eleven-twentieths of the share, equal to seven shares. Reduce this by removing the eleven-twentieths of a share, and adding them to the seven shares. Then you have the same four-fifths and one-tenth of one-fifth of capital, equal to seven shares and eleven-twentieths. Complete the capital by adding to any thing that you have nine forty-one parts. Then you have capital equal to nine shares and seventeen eighty-seconds. Now assume each portion to consist of eighty-two parts; then you have seven hundred and fifty-five parts. Two-fifths of these are three hundred (81) and two. Subtract from this the share of the daughter, which is eighty-two; there remain two hundred and twenty. Subtract from this one-fourth and one-fifth, namely, ninety-nine parts. There remain one hundred and twenty-one. Add to this three-fifths of the capital, namely, four hundred and fifty-three. Then you have five hundred and seventy-four, to be divided into seven shares, each of eighty-two parts. This is the share of the daughter; each son receives twice as much.

If the heirs are the same, and he bequeaths to a person as much as the share of a son, less one-fourth and one-fifth of what remains of two-fifths (of the capital) after the deduction of the share; then you see that this legacy is likewise determined by two-fifths. Subtract two shares (of a daughter) from them, since every son receives two (such) shares; there remain
two-fifths of the capital less two (such) shares. Add thereto what was excepted from the legacy, namely, one-fourth and one-fifth of the two-fifths less nine-tenths of (a daughter's) share.* Then you have two-fifths and nine-tenths of one-fifth of the capital less two (daughter's) shares and nine-tenths. Add to this three-fifths of the capital. Then you have one capital and nine-tenths of one-fifth of the capital less two (daughter's) shares and nine-tenths, equal to seven (such) shares. Reduce this by removing the two shares and nine-tenths and adding them to the seven shares. Then you have one capital and nine-tenths of one-fifth of the capital, equal to nine shares of a daughter and nine-tenths. Reduce this to one entire capital, by deducting nine fifty-ninths from what you have. There remains the capital equal to eight such shares and twenty-three fifty-ninths. Assume now each share (of a daughter) to contain fifty-nine parts. Then the whole heritage comprizes four hundred and ninety-five parts. Two-fifths of this are one hundred and ninety-eight

---

* \( v = \frac{1}{4} \) of the residue = a daughter's share.

\[
2v = \text{a son's share}
\]

\[
1 - 2v + \frac{9}{20} \left[ \frac{2}{5} - 2v \right] = 7v
\]

\[
i.e. \frac{3}{5} + \frac{2}{5} - 2v + \frac{9}{20} \left[ \frac{2}{5} - 2v \right] = 7v
\]

\[
\therefore \frac{3}{5} + \frac{2}{5} \left[ \frac{2}{5} - 2v \right] = 7v
\]

\[
\therefore \frac{3}{5} + \frac{2}{5} \cdot \frac{9}{10} = [7 + \frac{9}{10}]v \quad \therefore \frac{59}{5} = 99v
\]

\[
\therefore v = \frac{59}{495} \text{; a son's share} = \frac{113}{495}
\]

and the legacy to the stranger = \( \frac{82}{495} \)
parts. Subtract therefrom the two shares (of a daughter) or one hundred and eighteen parts; there remain eighty parts. Subtract now that which was excepted, namely, one-fourth and one fifth of these eighty, or thirty-six parts; there remain for the legatee eighty-two parts. Deduct this from the parts in the total number of parts in the heritage, namely, four hundred and ninety-five. There remain four hundred and thirteen parts to be distributed into seven shares; the daughter receiving (one share or) fifty-nine (parts), and each son twice as much.

If he leaves two sons and two daughters, and bequeaths to some person as much as the share* of a

* Since there are two sons and two daughters, each son receives \( \frac{1}{3} \), and each daughter \( \frac{1}{6} \) of the residue. Let \( v = \) a daughter's share.

Let the 1st legacy = \( x = v - \frac{1}{3} \left[ \frac{3}{3} - v \right] \)

\[ \begin{align*}
\text{and } 3d & \quad \ldots \ldots \quad = \frac{1}{12} \\
1 - \frac{1}{12} - x - y &= 6v \\
i.e. \quad \frac{3}{3} - \frac{1}{12} + \frac{1}{3} - x - v + \frac{1}{3} \left[ \frac{3}{3} - v \right] &= 6v \\
o r \quad \frac{3}{3} - \frac{1}{12} + \frac{1}{3} \left[ \frac{3}{3} - v \right] &= 6v \\
i.e. \quad \frac{7}{12} + \frac{3}{3} \left[ \frac{1}{3} - v + \frac{1}{3} \left[ \frac{3}{3} - v \right] - v \right] &= 6v \\
o r \quad \frac{1}{12} + \frac{3}{3} \left[ \frac{3}{3} - v \right] - v &= 6v \\
or \quad \frac{1}{12} + \frac{8}{3} = \left[ 6 + \frac{4}{5} \times \frac{11}{5} \right] v = \frac{13}{3} \frac{4}{3} v \\
or \quad \frac{1}{3} + \frac{8}{3} = \frac{13}{5} a \quad \therefore v = \frac{67}{23} \frac{4}{3} = \frac{1}{6} \\
The \text{1st Legacy } = x = \frac{1}{12} \\
The \text{2d } \ldots \ldots \quad = y = \frac{1}{12} \\
A \text{son's share } = \frac{1}{4}
daughter less one-fifth of what remains from one-third after the deduction of that share; and to another person as much as the share of the other daughter less one-third of what remains from one-third after the deduction of all this; and to another person half one-sixth of his entire capital; then you observe that all these legacies are determined by the one-third. Take one-third of the capital, and subtract from it the share of a daughter; there remains one-third of the capital less one share. Add to this that which was excepted, namely, one-fifth of the one-third less one-fifth of the share: this gives one-third and one-fifth of one-third of 

(83) the capital less one and one-fifth portion. Subtract herefrom the portion of the second daughter; there remain one-third and one-fifth of one-third of the capital less two portions and one-fifth. Add to this that which was excepted; then you have one-third and three-fifths of one-third, less two portions and fourteen-fifteenths of a portion. Subtract herefrom half one-sixth of the entire capital: there remain twenty-seven sixtieths of the capital less the two shares and fourteen-fifteenths, which are to be subtracted. Add thereto two-thirds of the capital, and reduce it, by removing the shares which are to be subtracted, and adding them to the other shares. You have then one and seven-sixtieths of capital, equal to eight shares and fourteen-fifteenths. Reduce this to one capital by subtracting from every thing that you have seven-sixtieths. Then let a share be two hundred
and one;* the whole capital will be one thousand six hundred and eight.

If the heirs are the same, and he bequeaths to a person as much as the share of a daughter, and one-fifth of what remains from one-third after the deduction of that share; and to another as much as the share of the second daughter and one-third of what remains from one-fourth after the deduction of that share: then, in the computation,† you must consider that the two legacies are determined by one-fourth and one-third. Take one-third of the capital, and subtract from it one share; there remains one-third of the capital less one share. Then subtract one-fifth of the remainder, namely, one-fifth of one-third of the capital, less one-fifth of the share; there remain four-fifths of one-third, less four-fifths of the share. Then take also one-fourth of the capital, and subtract from it one (84) share; there remains one-fourth of the capital, less one share. Subtract one-third of this remainder: there

\[ \frac{901}{1608} = \frac{1}{8} = \frac{3}{24} = y; \text{ and } \frac{1}{12} = \frac{3}{24} = x \]

The common denominator 1608 is unnecessarily great.

† Let \( x \) be the 1st legacy; \( y \) the 2d; \( v \) a daughter's share.

\[ 1 - x - y = 6v \]

\[ x = v + \frac{1}{3} \left[ \frac{1}{3} - v \right] \]

\[ y = v + \frac{1}{3} \left[ \frac{1}{4} - v \right] \]

Then \[ 1 - \frac{1}{3} - \frac{1}{4} + \frac{1}{3} - v - \frac{1}{3} \left[ \frac{1}{3} - v \right] + \frac{1}{4} - v - \frac{1}{3} \left[ \frac{1}{4} - v \right] = 6v \]

or \[ \frac{5}{12} + \frac{1}{3} - v + \frac{1}{3} - v = 6v \]

\[ \therefore \frac{5}{12} + \frac{1}{12} = 6v \]

\[ \therefore \frac{5}{12} + \frac{1}{3} + \frac{1}{2} = [6 + \frac{1}{3} + \frac{1}{2}] v \]

\[ \therefore \frac{5}{4} = \frac{1}{12} \cdot 2v ; \; \therefore \frac{5}{4} \cdot 4 \frac{1}{4} = \frac{15}{16} \]

\[ x = \frac{21}{44} ; \; y = \frac{21}{44} \]

\[ \frac{21}{44} \]
remain two-thirds of one-fourth of the capital, less two-thirds of one share. Add this to the remainder from the one-third of the capital; the sum will be twenty-six sixtieths of the capital, less one share and twenty-eight sixtieths. Add thereto as much as remains of the capital after the deduction of one-third and one-fourth from it; that is to say, one-fourth and one-sixth; the sum is seventeen-twentieths of the capital, equal to seven shares and seven-fifteenths. Complete the capital, by adding to the portions which you have three-seventeenths of the same. Then you have one capital, equal to eight shares and one-hundred-and-twenty hundred-and-fifty-thirds. Assume now one share to consist of one-hundred-and-fifty-three parts, then the capital consists of one thousand three hundred and forty-four. The legacy determined by one-third, after the deduction of one share, is fifty-nine; and the legacy determined by one-fourth, after the deduction of the share, is sixty-one.

If he leaves six sons, and bequeaths to a person as much as the share of a son and one-fifth of what remains of one-fourth; and to another person as much as the share of another son less one-fourth of what remains of one-third, after the deduction of the two first legacies and the second share; the computation is this:* You subtract one share from one-fourth of the capital;

* Let $x$ be the legacy to the 1st stranger
and $y \ldots \ldots \ldots \ldots \ldots 2d \ldots \ldots$; $v = \text{a son’s share}$
there remains one-fourth less the share. Remove then \((85)\) one-fifth of what remains of the one-fourth, namely, half one-tenth of the capital less one-fifth of the share. Then return to the one-third, and deduct from it half one-tenth of the capital, and four-fifths of a share, and one other share besides. The remainder then is one-third, less half one-tenth of the capital, and less one share and four-fifths. Add hereto one-fourth of the remainder, which was excepted, and assume the one-third to be eighty; subtracting from it half one-tenth of the capital, there remain of it sixty-eight less one share and four-fifths. Add to this one-fourth of it, namely, seventeen parts, less one-fourth of the shares to be subtracted from the parts. Then you have eighty-five parts less two shares and one-fourth. Add this to the other two-thirds of the capital, namely, one hundred and sixty parts. Then you have one and one-eighth of one-sixth of capital, less two shares and one-fourth, equal to six shares. Reduce this, by removing the shares which are to be subtracted, and adding

\[
1 - x - y = 6v
\]

\[
x = v + \frac{1}{4} \left[ \frac{1}{2} - v \right]; \quad y = v - \frac{1}{4} \left[ \frac{1}{3} - x - v \right]
\]

i.e. \[
\frac{3}{2} + \frac{1}{3} - x - v + \frac{1}{4} \left[ \frac{1}{3} - x - v \right] = 6v
\]

or \[
\frac{3}{2} + \frac{1}{4} \left[ \frac{1}{3} - x - v \right] = 6v
\]

or \[
\frac{3}{2} + \frac{1}{4} \left[ \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - v - \frac{1}{4} \right] - v = 6v
\]

or \[
\frac{3}{2} + \frac{5}{4} \left[ \frac{1}{12} + \frac{3}{4} \left[ \frac{1}{4} - v \right] - v \right] = 6v
\]

\[
\vdots \quad \frac{3}{2} + \frac{5}{4} \times \frac{1}{12} + \frac{1}{4} = [7 + \frac{5}{4}] v
\]

\[
\vdots \quad \frac{3}{2} + \frac{5}{12} + 1 = 33v \quad \vdots \quad \frac{49}{19} \times \frac{3}{3} = \frac{49}{3} = v
\]

\[
\vdots \quad x = v + \frac{10}{396}, \text{ and } y = v - \frac{6}{396}
\]
them to the other shares. Then you have one and one-eighth of one-sixth of capital, equal to eight shares and one-fourth. Reduce this to one capital, by subtracting from the parts as much as one forty-ninth of them. Then you have a capital equal to eight shares and four forty-ninths. Assume now every share to be forty-nine; then the entire capital will be three hundred and ninety-six; the share forty-nine; the legacy determined by one-fourth, ten; and the exception from the second share will be six.

On the Legacy with a Dirhem.

"A man dies, and leaves four sons, and bequeaths to some one a dirhem, and as much as the share of a son, and one-fourth of what remains from one-third after the deduction of that share." Computation:* Take

* Let the capital = $1$; a dirhem = $\delta$;
the legacy = $x$; and a son's share = $v$

$$1 - x = 4v$$

$$x = v + \frac{1}{4} \left( \frac{1}{3} - v \right) + \delta$$

$$\therefore \frac{2}{3} + \frac{1}{3} - v - \frac{1}{4} \left( \frac{1}{3} - v \right) - \delta = 4v$$

$$\therefore \frac{2}{3} + \frac{3}{4} \left[ \frac{1}{3} - v \right] - \delta = 4v$$

$$\therefore \frac{2}{3} + \frac{3}{4} - \delta = \left[4 + \frac{3}{4}\right] v$$

$$\therefore \frac{1}{12} - \delta = \frac{1}{4} \delta$$

$$\therefore \frac{11}{12}$$ of the capital $-\frac{12}{5}$ of a dirhem $= v$
and $\frac{11}{12}$ of the capital $+\frac{3}{4}$ of a dirhem $= x$, the legacy.

If we assume the capital to be so many dirhems, or a dirhem to be such a part of the capital, we shall obtain the
one-third of the capital and subtract from it one share; there remains one-third, less one share. Then subtract one-fourth of the remainder, namely, one-fourth of one-third, less one-fourth of the share; then subtract also one dirhem; there remain three-fourths of one-third of the capital, that is, one-fourth of the capital, less three-fourths of the share, and less one dirhem. Add this to two-thirds of the capital. The sum is eleven-twelfths of the capital, less three-fourths of the share and less one dirhem, equal to four shares. Reduce this by removing three-fourths of the share and one dirhem; then you have eleven-twelfths of the capital, equal to four shares and three-fourths, plus one dirhem. Complete your capital, by adding to the shares and one dirhem one-eleventh of the same. Then you have the capital equal to five shares and two-elevenths and one dirhem and one-eleventh. If you (87) wish to exhibit the dirhem distinctly, do not complete your capital, but subtract one from the eleven on account of the dirhem, and divide the remaining ten by the portions, which are four and three-fourths. The quotient is two and two-nineteenth of a dirhem. Assuming, then, the capital to be twelve dirhems, each

\[ v = \frac{1}{3} \left( 11 - 1 \right) = \frac{1}{3} \cdot 10 = \frac{10}{3} \text{ dirhems,} \]
\[ x = \frac{1}{5} \left( 13 + 4 \right) = \frac{2}{5} \cdot 17 = \frac{34}{5} \text{ dirhems.} \]
share will be two dirhems and two-nineteenth. Or, if you wish to exhibit the share distinctly, complete your square, and reduce it, when the dirhem will be eleven of the capital.

If he leaves five sons, and bequeath to some person a dirhem, and as much as the share of one of the sons, and one-third of what remains from one-third, and again, one-fourth of what remains from the one-third after the deduction of this, and one dirhem more; then the computation is this:* You take one-third, and subtract one share; there remains one-third less one share. Subtract herefrom that which is still in your hands, namely, one-third of one-third less one-third of the share. Then subtract also the dirhem; there remain two-thirds of one-third, less two-thirds of the share and less one dirhem. Then subtract one-fourth of what you have, that is, one-eighteenth, less one-sixth of a share and less one-fourth of a dirhem, and

* Let the legacy = x; and a son’s share = v

\[ 1 - x = 5v \]
\[ \frac{3}{8} + \frac{1}{3} - v - \frac{3}{8} \left[ \frac{3}{8} - v \right] - \frac{3}{8} = 5v \]
i.e. \[ \frac{3}{8} + \frac{3}{8} \left[ \frac{3}{8} - v \right] - \frac{3}{8} = 5v \]
i.e. \[ \frac{3}{8} + \frac{3}{8} \left[ \frac{3}{8} - v \right] - \frac{3}{8} = 5v \]
\[ \vdots \]
\[ \frac{3}{8} + \frac{1}{3} - \frac{1}{3}v - \frac{3}{8} = 5v \]
\[ \vdots \]
\[ \frac{3}{8} + \frac{1}{3}v = \frac{3}{8} \]
\[ \vdots \]
\[ \frac{2}{5} \text{ of the capital} - \frac{2}{5} \text{ of a dirhem} = v \]
\[ \vdots \]
\[ \frac{10}{9} \text{ of the capital} + \frac{10}{9} \text{ of a dirhem} = x, \text{ the legacy.} \]

If the capital = \( \frac{4}{5} \) dirhems, or \( \frac{1}{3} \) of the capital = \( 7\frac{1}{2} \) dirhems,

\[ v = \frac{4}{11} \text{ dirhems} = 3\frac{1}{11} \text{ dirhems.} \]
subtract also the second dirhem; the remainder is half one-third of the capital, less half a share and less one dirhem and three-fourths; add thereto two-thirds of the capital, the sum is five-sixths of the capital, less one half of a share, and less one dirhem and three-fourths, equal to five shares. Reduce this, by removing the half share and the one dirhem and three-fourths, and adding them to the (five) shares. Then you have five-sixths of capital, equal to five shares and a half plus one dirhem and three-fourths. Complete your capital, by adding to five shares and a half and to one dirhem and three-fourths, as much as one-fifth of the same. Then you have the capital equal to six shares and three-fifths plus two dirhems and one-tenth. Assume, now, each share to consist of ten parts, and one dirhem likewise of ten; then the capital is eighty-seven parts. Or, if you wish to exhibit the dirhem distinctly, take the one-third, and subtract from it the share; there remains one-third, less one share. Assume the one-third (of the capital) to be seven and a half (dirhems). Subtract one-third of what you have, namely, one-third of one-third;* there remain two-thirds of one-third, less two-thirds of the share: that is, five dirhems, less two-thirds of the share. Then subtract one, on account of the one dirhem, and you retain four dirhems, less two-thirds

* There is an omission here of the words "less one third of a share."
of the share. Subtract now one-fourth of what you have, namely, one part less one-sixth of a share; and remove also one part on account of the one dirhem; the remainder, then, is two parts less half a share. Add this to the two-thirds of the capital, which is fifteen (dirhems). Then you have seventeen parts less half a share, equal to five shares. Reduce this, by removing half a share, and adding it to the five shares. Then it is seventeen parts, equal to five shares and a half. Divide now seventeen by five and a half; the quotient is the value of one share, namely, three dirhems and one-eleventh; and one-third (of the capital) is seven and a half (dirhems).

If he leaves four sons, and bequeaths to some person as much as the share of one of his sons, less one-fourth of what remains from one-third after the deduction of the share, and one dirhem; and to another one-third of what remains from the one-third, and one dirhem; then this legacy is determined by one-third.*

* Let the 1st legacy be \( x \), the 2d \( y \); and a son's share = \( v \)

\[
1 - x - y = 4v
\]

i.e. \( \frac{2}{3} + \frac{1}{3} - v + \frac{1}{4} \left[ \frac{1}{3} - v \right] = \frac{2}{3} \left[ \frac{1}{3} - v + \frac{1}{4} \left( \frac{1}{3} - v \right) \right] - \frac{2}{3} = 4v \)

i.e. \( \frac{5}{3} + \frac{5}{3} \left[ \frac{1}{3} - v \right] + \frac{1}{4} \left[ \frac{1}{3} - v \right] = 4v \)

i.e. \( \frac{5}{3} + \frac{5}{3} \left[ \frac{1}{3} - v \right] - \frac{2}{3} = 4v \)

\[
\therefore \frac{5}{3} + \frac{5}{3} - \frac{5}{3} v - \frac{5}{3} \frac{2}{3} = 4v
\]

\[
\therefore \frac{17}{18} - \frac{5}{3} \frac{2}{3} = \frac{2}{3} v
\]

\[
\therefore \frac{17}{18} - \frac{5}{3} \frac{2}{3} = v
\]

also \( \frac{17}{18} + \frac{3}{3} \frac{2}{3} \frac{2}{3} = x \)

\[
\frac{5}{3} + \frac{2}{3} \frac{2}{3} \frac{2}{3} = y
\]
Take one-third of the capital, and subtract from it one share; there remains one-third, less one share; add hereto one-fourth of what you have: then it is one-third and one-fourth of one-third, less one share and one-fourth. Subtract one dirhem; there remains one-third of one and one-fourth, less one dirhem, and less one share and one-fourth. There remains from the one-third as much as five-eighteenths of the capital, less two-thirds of a dirhem, and less five-sixths of a share. Now subtract the second dirhem, and you retain five-eighteenths of the capital, less one dirhem and two-thirds, and less five-sixths of a share. Add to this two-thirds of the capital, and you have seventeen-eighteenths of the capital, less one dirhem and two-thirds, and less five-sixths of a share, equal to four shares. Reduce this, by removing the quantities which are to be subtracted, and adding them to the shares; then you have seventeen-eighteenths of the capital, equal to four portions and five-sixths plus one dirhem and two-thirds. Complete your capital by (90) adding to the four shares and five-sixths, and one dirhem and two-thirds, as much as one-seventeenth of the same. Assume, then, each share to be seventeen, and also one dirhem to be seventeen.* The whole capital will then be one hundred and seventeen. If you wish to exhibit the dirhem distinctly, proceed with it as I have shown you.

* Capital = \( \frac{3}{17} \frac{v}{v} + \frac{3}{9} \) ... if \( v = 17 \), and \( \delta = 17 \), capital = 117
If he leaves three sons and two daughters, and bequeaths to some person as much as the share of a daughter plus one dirhem; and to another one-fifth of what remains from one-fourth after the deduction of the first legacy, plus one dirhem; and to a third person one-fourth of what remains from one-third after the deduction of all this, plus one dirhem; and to a fourth person one-eighth of the whole capital, requiring all the legacies to be paid off by the heirs generally: then you calculate this by exhibiting the dirhems distinctly, which is better in such a case.* Take one-fourth of the capital, and assume it to be six dirhems; the entire capital will be twenty-four dirhems. Subtract one share from the one-fourth; there remain six dirhems less one share. Subtract also one dirhem; there remain five dirhems less one share. Subtract

* Let the legacies to the three first legatees be, severally, \(x, y, z\); the fourth legacy = \(\frac{1}{3}\); and let a daughters' share = \(v\).

\[
\begin{align*}
&\therefore \frac{7}{3} - x - y - z = 8v \\
x &= v + \delta; \quad y = \frac{2}{3} \left[\frac{1}{3} - x\right] + \delta; \quad z = \frac{4}{3} \left[\frac{1}{3} - x - y\right] + \delta \\
&\text{Then } \frac{7}{3} - \frac{1}{3} + \frac{1}{3} - x - y - \frac{4}{3} \left[\frac{1}{3} - x - y\right] - \delta = 8v \\
&\therefore \frac{1}{3} + \frac{4}{3} \left[\frac{1}{3} - x - y\right] - \delta = 8v \\
&\text{but } \frac{1}{3} - x - y = \frac{1}{3} - \frac{1}{3} + \frac{1}{3} - x - \frac{1}{3} \left[\frac{1}{3} - x\right] - \delta \\
&= \frac{1}{3} + \frac{4}{3} \left[\frac{1}{3} - x\right] - \delta \\
&= \frac{1}{3} + \frac{4}{3} \frac{3}{5} v - \frac{2}{3} \delta \\
&= \frac{1}{3} - \frac{3}{5} v - \frac{2}{3} \delta \\
&\therefore \frac{1}{3} + \frac{2}{3} \times \frac{3}{5} - \frac{7}{5} - \frac{4}{3} \left[\frac{1}{3} - x\right] - \delta = 8v \\
&\therefore \frac{181}{200} - \frac{47}{200} \delta = \frac{43}{5} v \quad v = \frac{1811}{2000} - \frac{564}{2000} \delta, \text{ and } 1 = \frac{2064}{1811} v + \frac{564}{1811} \delta \\
x = \frac{1811}{2000} + \frac{564}{2000} \delta; \quad y = \frac{67}{2000} + \frac{1764}{2000} \delta; \quad z = \frac{110}{2000} + \frac{1248}{2000} \delta
\end{align*}
\]
one-fifth of this remainder; there remain four dirhems, less four-fifths of a share. Now deduct the second dirhem, and you retain three dirhems, less four-fifths of a share. You know, therefore, that the legacy which was determined by one-fourth, is three dirhems, less four-fifths of a share. Return now to the one-third, which is eight, and subtract from it three dirhems, less four-fifths of a share. There remain five dirhems, less four-fifths of a share. Subtract also one-fourth of this and one dirhem, for the legacy; you then retain two dirhems and three-fourths, less three-fifths of a share. Take now one-eighth of the capital, namely, three; after the deduction of one-third, you retain one-fourth of a dirhem, less three-fifths of a share. Return now to the two-thirds, namely, sixteen, and subtract from them one-fourth of a dirhem less three-fifths of a share; there remain of the capital fifteen dirhems and three-fourths, less three-fifths of a share, which are equal to eight shares. Reduce this, by removing three-fifths of a share, and adding them to the shares, which are eight. Then you have fifteen dirhems and three-fourths, equal to eight shares and three-fifths. Make the division: the quotient is one share of the whole capital, which is twenty-four (dirhems). Every daughter receives one dirhem and one-hundred-and-forty-three one-hundred-and-seventy-second parts of a dirhem.*

* $v = \frac{181}{2063}$ of the capital $- \frac{564}{2063}$ of a dirhem. If we assume
If you prefer to produce the shares distinctly, take one-fourth of the capital, and subtract from it one share; there remains one-fourth of the capital less one share. Then subtract from this one dirhem: then subtract one-fifth of the remainder of one-fourth, which is one-fifth of one-fourth of the capital, less one-fifth of the share and less one-fifth of a dirhem; and subtract also the second dirhem. There remain four-fifths of the one-fourth less four-fifths of a share, and less one dirhem and four-fifths. The legacies paid out of one fourth amount to twelve two-hundred-and-fortieths of the capital and four-fifths of a share, and one dirhem and four-fifths. Take one-third, which is eighty, and subtract from it twelve, and four-fifths of a share, and one dirhem and four-fifths, and remove one-fourth of what remains, and one dirhem. You retain, then, of the one-third, only fifty-one, less three-fifths of a share, less two dirhems and seven-twentieths. Subtract herefrom one-eighth of the capital, which is thirty, and you retain twenty-one, less three-fifths of a share, and less two dirhems and seven-twentieths, and two-thirds of the capital, being equal to eight shares. Reduce this, by removing that which is to be subtracted, and adding it to the eight shares. Then you have one hundred and eighty-one parts of the capital to be equal to 24 dirhems

\[ v = \frac{181 \times 24 - 564}{2064} \text{ dirhems} = \frac{4344 - 564}{2064} \]

\[ = \frac{3780}{2064} \text{ = } 1 \frac{145}{2064} \text{ dirhems.} \]
capital, equal to eight shares and three-fifths, plus two dirhems and seven twentieths. Complete your capital, by adding to that which you have fifty-nine one-hundred-and-eighty-one parts. Let, then, a share be three hundred and sixty-two, and a dirhem likewise three hundred and sixty-two.* The whole capital is then five thousand two hundred and fifty-six, and the legacy out of one-fourth† is one thousand two hundred and four, and that out of one-third is four hundred and ninety-nine, and the one-eighth is six hundred and fifty-seven.

On Completement.

"A woman dies and leaves eight daughters, a mother, and her husband, and bequeaths to some person as much as must be added to the share of a daughter to make it equal to one-fifth of the capital; and to another person as much as must be added to the share of the mother to make it equal to one-fourth of

* The capital $= \frac{2064}{181} \cdot v + \frac{564}{181} \cdot d$

If we assume $v = 362$, and $d = 362$, the capital $= 5256$

Then $x = 724; y = 480; z = 499; \frac{1}{8}$th of capital $= 657$.

† The text ought to stand "the two first legacies are" instead of "the legacy out of one-fourth is."

The first legacy is ............... 724
The second ....................... 480

... the first + second legacy = 1204
the capital.”* Computation: Determine the parts of the residue, which in the present instance are thirteen. Take the capital, and subtract from it one-fifth of the same, less one part, as the share of a daughter: this being the first legacy. Then subtract also one-fourth, less two parts, as the share of the mother: this being the second legacy. There remain eleven-twentieths of the capital, which, when increased by three parts, are equal to thirteen parts. Remove now from thirteen parts the three parts on account of the three parts (on the other side), and you retain eleven-twentieths of the capital, equal to ten parts. Complete the capital, by adding to the ten parts as much as nine-elevenths of the same; then you find the capital equal to eighteen parts and two-elevenths. Assume now each part to be eleven; then the whole capital is two hundred, each part is eleven; the first legacy will be twenty-nine, and the second twenty-eight.

If the case is the same, and she bequeaths to some person as much as must be added to the share of the husband to make it equal to one-third, and to another person as much as must be added to the share of the mother to make it equal to one-fourth; and to a

* In this case, the mother has \( \frac{2}{13} \); and each daughter has \( \frac{1}{13} \) of the residue.

\[
1 - x - y = 13v
\]

i.e. \( 1 - \frac{1}{3} + v - \frac{1}{4} + 2v = 13v \)

\[
\therefore \frac{11}{20} = 10v \quad \therefore v = \frac{11}{200} \quad x = \frac{9}{200} \quad y = \frac{98}{200}
\]
third as much as must be added to the share of a daughter to make it equal to one-fifth; all these legacies being imposed on the heirs generally: then you divide the residue into thirteen parts. Take the capital, and subtract from it one-third, less three parts, being the share of the husband; and one-fourth, less two parts, being the share of the mother; and lastly, one-fifth less one part, being the share of a daughter. The remainder is thirteen-sixtieths of the capital, which, when increased by six parts, is equal to thirteen parts. Subtract the six from the thirteen parts: there remain thirteen-sixtieths of the capital, equal to seven parts. Complete your capital by multiplying the seven parts by four and eight-thirteen, and you have a capital equal to thirty-two parts and four-thirteenths.

Assuming then each part to be thirteen, the whole capital is four hundred and twenty.

If the case is the same, and she bequeaths to some person as much as must be added to the share of the mother to make it one-fourth of the capital; and to another as much as must be added to the portion of a daughter, to make it one-fifth of what remains of the capital, after the deduction of the first legacy; then

\[
* \quad 1 - \left[ \frac{1}{3} - 3v \right] - \left[ \frac{1}{4} - 2v \right] - \left[ \frac{1}{5} - v \right] = 13v
\]

i.e. \[
1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} = 7v
\]

\[
\therefore \quad \frac{13}{60} = 7v
\]

\[
\therefore \quad v = \frac{13}{420}
\]
you constitute the parts of the residue by taking them out of thirteen.* Take the capital, and subtract from it one-fourth less two parts; and again, subtract one-fifth of what you retain of the capital, less one part; then look how much remains of the capital after the deduction of the parts. This remainder, namely, three-fifths of the capital, when increased by two parts and three-fifths, will be equal to thirteen parts. Subtract two parts and three-fifths from thirteen parts, there remain ten parts and two-fifths, equal to three-fifths of capital. Complete the capital, by adding to the parts which you have, as much as two-thirds of the same. Then you have a capital equal to seventeen parts and one-third. Assume a part to be three, then the capital is fifty-two, each part three; the first legacy will be seven, and the second six.

If the case is the same, and she bequeaths to some person as much as must be added to the share of the mother to make it one-fifth of the capital, and to another one-sixth of the remainder of the capital; then

\[ 1 - x - y = 13v \]
\[ x = \frac{1}{2} - 2v; \quad y = \frac{1}{3}[1 - x] - v \]
\[ 1 - x - \frac{1}{3}[1 - x] + v = 13v \]
\[ \frac{3}{5} [1 - x] = 12v \quad \therefore \frac{3}{5} \left[ \frac{3}{5} + 2v \right] = 12v \]
\[ \therefore \frac{3}{5} = \left[ 12 - \frac{3}{5} \right] v = \frac{52}{3} v \]
\[ \therefore v = \frac{3}{52} \quad \therefore x = \frac{7}{52}, \; y = \frac{6}{52} \]
the parts are thirteen.* Take the capital, and subtract from it one-fifth less two parts; and again, subtract one-sixth of the remainder. You retain two-thirds of the capital, which, when increased by one part and two-thirds, are equal to thirteen parts. Subtract the one part and two-thirds from the thirteen parts: there remain two thirds of the capital, equal to eleven parts and one-third. Complete your capital, by adding to the parts as much as their moiety; thus you find the capital equal to seventeen parts. Assume now the capital to be eighty-five, and each part five; then the first legacy is seven, and the second thirteen, and the remaining sixty-five are for the heirs.

If the case is the same, and she bequeaths to some person as much as must be added to the share of the mother, to make it one-third of the capital, less that sum which must be added to make the share of a daughter equal to one-fourth of what remains of the capital after the deduction of the above complement; then the parts are thirteen.† Take the capital, and

\[
\begin{align*}
* \quad 1-x-y &= 13v \\
x &= \frac{1}{3} - 2v; \quad y = \frac{1}{6} [1-x] \\
1-x-\frac{1}{6} [1-x] &= 13v \\
\therefore \frac{5}{6} [1-x] &= 13v \\
\therefore \frac{5}{6} [\frac{1}{3} + 2v] &= 13v \\
\therefore \frac{5}{3} + \frac{5}{6} v &= 13v \\
\therefore \frac{5}{3} &= \frac{34}{3} v \quad \therefore v = \frac{1}{17}; \quad x = \frac{7}{5}; \quad y = \frac{13}{83} \\
\end{align*}
\]

† \[1-x+y = 13v; \quad \text{and } x = \frac{1}{3} - 2v; \quad y = \frac{1}{4} [1-x] - v \]

\[
\begin{align*}
\therefore 1-x+\frac{1}{4} [1-x] - v &= 13v \\
\therefore \frac{5}{4} [1-x] &= 14v \quad \therefore \frac{5}{4} [\frac{1}{3} + 2v] &= 14v \\
\therefore \frac{5}{6} &= \frac{23}{2} v \quad \therefore v = \frac{5}{3}; \quad x-y = \frac{4}{9} \\
\end{align*}
\]
subtract from it one-third less two parts, and add to the remainder one-fourth (of such remainder) less one part; then you have five-sixths of the capital and one part and a half, equal to thirteen parts. Subtract one part and a half from thirteen parts. There remain eleven parts and a half, equal to five-sixths of the capital. Complete the capital, by adding to the parts as much as one-fifth of them. Thus you find the capital equal to thirteen parts and four-fifths. Assume, now, a part to be five, then the capital is sixty-nine, and the legacy four.

"A man dies, and leaves a son and five daughters, and bequeaths to some person as much as must be added to the share of the son to complete one-fifth and one-sixth, less one-fourth of what remains of one-third after the subtraction of the complement."* Take one-third of the capital, and subtract from it one-fifth and one-sixth of the capital, less two (seventh) parts; so that you retain two parts less four one hundred and twentieths of the capital. Then add it to the exception, which is half a part less one one hundred and

* Since there are five daughters and one son, each daughter receives \( \frac{1}{5} \), and the son \( \frac{2}{7} \) of the residue.

\[
1 - x = 7v; \quad \frac{1}{3} + \frac{1}{6} = \frac{1}{3}
\]

\[
\therefore \frac{2}{3} + \frac{1}{3} - \frac{1}{3} + 2v + \frac{1}{3} \left[ \frac{1}{3} - \frac{1}{3} + 2v \right] = 7v
\]

\[
\therefore \frac{2}{3} - \frac{1}{3} + 2v + \frac{1}{3} \left[ \frac{2}{3} + \frac{1}{3} \right] = 7v
\]

\[
\therefore \frac{2}{3} + \frac{5}{6} \left[ \frac{2}{3} + 2v \right] = 7v
\]

\[
\therefore \frac{4}{3} - \frac{1}{3} = \frac{9}{2} v
\]

\[
\therefore \frac{9}{2} = \frac{9}{2} v \quad \therefore v = \frac{5}{36}, \text{ and } x = \frac{3}{5}
\]
twentieth, and you have two parts and a half less five
one hundred and twentieths of capital. Add hereto
two-thirds of the capital, and you have seventy-five
one hundred and twentieths of the capital and two
parts and a half, equal to seven parts. Subtract, now,
two parts and a half from seven, and you retain seventy-
five one hundred and twentieths, or five-eighths, equal
to four parts and a half. Complete your capital, by (97)
adding to the parts as much as three-fifths of the same,
and you find the capital equal to seven parts and one-
fifth part. Let each part be five; the capital is then
thirty-six, each portion five, and the legacy one.

If he leaves his mother, his wife, and four sisters,
and bequeaths to a person as much as must be added to
the shares of the wife and a sister, in order to make them
equal to the moiety of the capital, less two-sevenths of
the sum which remains from one-third after the deduc-
tion of that complement; the Computation is this:* If

* From the context it appears, that when the heirs of the
residue are a mother, a wife, and 4 sisters, the residue is to
be divided into 13 parts, of which the wife and one sister,
together, take 5: therefore the mother and 3 sisters, toget-
ther, take 8 parts. Each sister, therefore, must take not
less than \(\frac{1}{13}\), nor more than \(\frac{2}{13}\). In the case stated at page
102, a sister was made to inherit as much as a wife; in the
present case that is not possible; but the widow must take
not less than \(\frac{3}{13}\); and each sister not more than \(\frac{2}{13}\). Prob-
bly, in this case, the mother is supposed to inherit \(\frac{3}{13}\); the
wife \(\frac{3}{13}\); each sister \(\frac{2}{13}\).
you take the moiety from one-third, there remains one-sixth. This is the sum excepted. It is the share of the wife and the sister. Let it be five (thirteenth) parts. What remains of the one-third is five parts less one-sixth of the capital. The two-sevenths which he has excepted are two-sevenths of five parts less two-sevenths of one-sixth of the capital. Then you have six parts and three-sevenths, less one-sixth and two-sevenths of one-sixth of the capital. Add hereto two-thirds of the capital; then you have nineteen forty-seconds of the capital and six parts and three-sevenths, equal to thirteen parts. Subtract herefrom the six parts and three-sevenths. There remain nineteen forty-seconds of the capital, equal to six parts and four-sevenths. Complete your capital by adding to it its double and four-nineteenths of it. 

\[
\begin{align*}
x + 5v &= \frac{1}{3}; \quad 1 - x + \frac{2}{9} \left[ \frac{1}{3} - x \right] = 13v \\
\therefore \frac{2}{3} + \frac{2}{9} \left[ \frac{1}{3} - x \right] &= 13v \\
\therefore \frac{2}{3} + \frac{2}{9} \left[ \frac{1}{3} - x \right] &= 13v \\
\therefore \frac{2}{3} + \frac{2}{9} \left[ \frac{-5}{9} + 5v \right] &= 13v \\
\therefore \frac{2}{3} - \frac{3}{13} &= \left[ 13 - \frac{45}{7} \right] v \\
\therefore \frac{10}{2} = \frac{45}{7} v \quad \therefore \frac{10}{2} = v \\
\therefore x = \frac{2}{2} v, \text{ and the residue } = \frac{9}{2} v \\
\end{align*}
\]

The author unnecessarily takes \(7 \times 276 = 1932\) for the common denominator.
two; each part is one hundred and thirty-three, the completion of it is three hundred and one, and the exception of one-third is ninety-eight, so that the remaining legacy is two hundred and three. For the heirs remain one thousand seven hundred and twenty-nine.

COMPUTATION OF RETURNS.*

On Marriage in Illness.

"A man, in his last illness, marries a wife, paying (a marriage settlement of) one hundred dirhems, besides which he has no property, her dowry being

* The solutions which the author has given of the remaining problems of this treatise, are, mathematically considered, for the most part incorrect. It is not that the problems, when once reduced into equations, are incorrectly worked out; but that in reducing them to equations, arbitrary assumptions are made, which are foreign or contradictory to the data first enounced, for the purpose, it should seem, of forcing the solutions to accord with the established rules of inheritance, as expounded by Arabian lawyers.

The object of the lawyers in their interpretations, and of the author in his solutions, seems to have been, to favour heirs and next of kin; by limiting the power of a testator, during illness, to bequeath property, or to emancipate slaves; and by requiring payment of heavy ransom for slaves whom a testator might, during illness, have directed to be emancipated.
ten dirhems. Then the wife dies, bequeathing one-third of her property. After this the husband dies.*

Computation: You take from the one hundred that which belongs entirely to her, on account of the dowry, namely, ten dirhems; there remain ninety dirhems, out of which she has bequeathed a legacy. Call the sum given to her (by her husband, exclusive of her dowry) thing; subtracting it, there remain ninety dirhems less thing. Ten dirhems and thing are already in her hands; she has disposed of one-third of her property, which is three dirhems and one-third, and one-third of thing; there remain six dirhems and

* Let $s$ be the sum, including the dowry, paid by the man, as a marriage settlement; $d$ the dowry; $x$ the gift to the wife, which she is empowered to bequeath if she pleases.

She may bequeath, if she pleases, $d+x$; she actually does bequeath $\frac{1}{3}[d+x]$; the residue is $\frac{2}{3}[d+x]$, of which one half, viz. $\frac{1}{3}[d+x]$ goes to her heirs, and the other half reverts to the husband.

:. the husband's heirs have $s-[d+x]+\frac{1}{3}[d+x]$ or $s-\frac{2}{3}[d+x]$; and since what the wife has disposed of, exclusive of the dowry, is $x$, twice which sum the husband is to receive, $s-\frac{2}{3}[d+x]=2x$. But $s=100;\ d=10\ \therefore x=35;\ d+x=45;\ \frac{1}{3}[d+x]=15$. Therefore the legacy which she bequeaths is 15, her husband receives 15, and her other heirs, 15. The husband's heirs receive $2x=70$.

But had the husband also bequeathed a legacy, then, as we shall see presently, the law would have defeated, in part, the woman's intentions.
two-thirds plus two-thirds of thing, the moiety of
which, namely, three dirhems and one-third plus one-
third of thing, returns as his portion to the husband.*
Thus the heirs of the husband obtain (as his share)
ninety-three dirhems and one-third, less two-thirds of
thing; and this is twice as much as the sum given to (99)
the woman, which was thing, since the woman had
power to bequeath one-third of all which the husband
left;† and twice as much as the gift to her is two
things. Remove now the ninety-three and one-third,
from two-thirds of thing, and add these to the two
things. Then you have ninety-three dirhems and one-
third equal to two things and two-thirds. One thing is
three-eighths of it, namely, as much as three-eighths
of the ninety-three and one-third, that is, thirty-five
dirhems.

If the question is the same, with this exception only,
that the wife has ten dirhems of debts, and that she
bequeaths one-third of her capital; then the Computa-

* In other cases, as appears from pages 92 and 93, a
husband inherits one-fourth of the residue of his wife's es-
tate, after deducting the legacies which she may have
bequeathed. But in this instance he inherits half the re-
sidue. If she die in debt, the debt is first to be deducted
from her property, at least to the extent of her dowry (see
the next problem.)

† When the husband makes a bequest to a stranger, the
third is reduced to one-sixth. Vide p. 137.
tion is as follows:* Give to the wife the ten dirhems of her dowry, so that there remain ninety dirhems, out of which she bequeath a legacy. Call the gift to her thing; there remain ninety less thing. At the disposal of the woman is therefore ten plus thing. From this her debts must be subtracted, which are ten dirhems. She retains then only thing. Of this she bequeath one-third, namely, one-third of thing: there remains two-thirds of thing. Of this the husband receives by inheritance the moiety, namely, one-third of thing. The heirs of the husband obtain, therefore, ninety dirhems, less two-thirds of thing; and this is twice as much as the gift to her, which was thing; that is, two things. Reduce this, by removing the two-thirds of thing from ninety, and adding them to two things. Then you have ninety dirhems, equal to two things and two-thirds. One thing is three-eighths of this; that is to say, thirty-three dirhems and three-fourths, which is the gift (to the wife).

If he has married her, paying (a marriage settle-

* The same things being assumed as in the last example, 
\[ s - [d + x] \] remains with the husband; \( d \) goes to pay the debts of the wife; and \( \frac{x}{3} \) reverts from the wife to the husband.

\[ \therefore s - d - \frac{2}{3}x = 2x \quad \therefore \frac{3}{3} [s - d] = x \]

\[ \therefore \] if \( s = 100 \), and \( d = 10 \), \( x = \frac{33}{3} \); she bequeaths \( 11\frac{1}{2} \); \( 11\frac{1}{2} \) reverts to her husband; and her other heirs receive \( 11\frac{1}{4} \). The husband's heirs receive \( 2x = 67\frac{1}{2} \).
ment of one hundred dirhems, her dowry being ten (100) dirhems, and he bequeaths to some person one-third of his property; then the computation is this: * Pay to the woman her dowry, that is, ten dirhems; there remain ninety dirhems. Herefrom pay the gift to her, thing; then pay likewise to the legatee who is to receive one-third, thing: for the one-third is divided

* This case is distinguished from that in page 133 by two circumstances; first, that the woman does not make any bequest; second, that the husband bequeaths one-third of his property.

Suppose the husband not to make any bequest. Then, since the woman had at her disposal $d+x$, but did not make any bequest, $\frac{1}{3} [d+x]$ reverts to her husband; and the like amount goes to her other heirs.

$$s - \frac{1}{3} [d+x] = 2x$$

and since $s = 100$, and $d = 10$; $x = 38$; $d+x = 48$; $\frac{1}{3} [d+x] = 24$ reverts to the husband, and the like sum goes to her other heirs; and $2x = 76$, belongs to the husband's heirs.

Now suppose the husband to bequeath one-third of his property. The law here interferes with the testator's right of bequeathing; and provides that whatever sum is at the disposal of the wife, the same sum shall be at the disposal of the husband; and that the sum to be retained by the husband's heirs shall be twice the sum which the husband and wife together may dispose of.

$$s - \frac{1}{3} [d+x] - x = 4x$$

$$\frac{1}{3} [2s - d] = x; \quad \text{if } s = 100, \text{ and } d = 10; \quad x = \frac{100}{3} = 17 \frac{1}{3};$$

$$d+x = 27 \frac{3}{4}; \quad \frac{1}{3} [d+x] = 13 \frac{1}{3} \text{ reverts to the husband, and}$$

the like sum goes to the other heirs of the woman; $17 \frac{1}{3}$ is what the husband bequeaths; and $69 \frac{1}{3} = 4x$ goes to the husband's heirs.
into two moieties between them, since the wife cannot take any thing, unless the husband takes the same. Therefore give, likewise, to the legatee who is to have one-third, thing. Then return to the heirs of the husband. His inheritance from the woman is five dirhems and half a thing. There remains for the heirs of the husband ninety-five less one thing and a half, which is equal to four things. Reduce this, by removing one thing and a half, and adding it to the four things. There remain ninety-five, equal to five things and a half. Make them all moieties; there will be eleven moieties; and one thing will be equal to seventeen dirhems and three-elevenths, and this will be the legacy.

"A man has married a wife paying (a marriage settlement of) one hundred dirhems, her dowry being ten dirhems; and she dies before him, leaving ten dirhems, and bequeathing one-third of her capital; afterwards the husband dies, leaving one hundred and twenty dirhems, and bequeathing to some person one-third of his capital." Computation:* Give to the wife her dowry,

* Let $c$ be the property which the wife leaves, besides $d$ the dowry, and $x$ the gift from the husband. She bequeaths \( \frac{1}{3} \) \( [c+d+x] \); \( \frac{1}{3} \) \( [c+d+x] \) goes to her husband; and \( \frac{1}{3} \) \( [c+d+x] \) to her other heirs. The husband leaves property $s$, out of which must be paid the dowry, $d$; the gift to the wife, $x$; and the bequest he makes to the stranger, $x$; and his heirs receive from the wife's heirs \( \frac{1}{3} [c+d+x] \)
namely, ten dirhems; then one hundred and ten dirhems remain for the heirs of the husband. From these the (101) gift to the wife is thing, so that there remain one hundred and ten dirhems less thing; and the heirs of the woman obtain twenty dirhems plus thing. She bequeaths one-third of this, namely, six dirhems and two-thirds, and one-third of thing. The moiety of the residue, namely, six dirhems and two-thirds plus one-third of thing, returns to the heirs of the husband: so that one hundred and sixteen and two-thirds, less two-thirds of thing, come into their hands. He has bequeathed one-third of this, which is thing. There remain, therefore, one hundred and sixteen dirhems and two-thirds less one thing and two-thirds, and this is twice as much as the husband's gift to the wife added to his legacy to the stranger, namely, four things. Reduce this, and you find one hundred and sixteen dirhems and two-thirds, equal to five things and two-thirds. Consequently one thing is equal to

\[ s - d - 2x + \frac{1}{3} [c + d + x] = 4x, \]

according to the law of inheritance.

\[ 3s + c - 2d = 17x, \text{ and } x = \frac{3s + c - 2d}{17} \]

If \( s = 120, c = 10, \) and \( d = 10, \) \( x = \frac{3 \times 120 + 10 - 2 	imes 10}{17} = 20 \frac{10}{17} \)
\( c + d + x = 40 \frac{10}{17}; \quad \frac{1}{3} [c + d + x] = 13 \frac{2}{17} \)

The wife bequeaths 13\frac{2}{17}; 13\frac{2}{17} go to her husband, and 13\frac{2}{17} to her other heirs.

The husband bequeaths to the stranger 20\frac{10}{17}; he gives the same sum to the wife; and 4x = 82\frac{6}{17} go to his heirs.
twenty dirhems and ten-seventeenths; and this is the legacy.

On Emancipation in Illness.

"Suppose that a man on his death-bed were to emancipate two slaves; the master himself leaving a son and a daughter. Then one of the two slaves dies, leaving a daughter and property to a greater amount than his price." You take two-thirds of his price, and what the other slave has to return (in order to complete his ransom). If the slave die before the master, then the son and the daughter of the latter partake of the heritage, in such proportion, that the son receives as much as the two daughters together. But if the slave die after the master, then you take two-thirds of his value and what is returned by the other slave, and distribute

* From the property of the slave, who dies, is to be deducted and paid to the master's heirs, first, two-thirds of the original cost of that slave, and secondly what is wanting to complete the ransom of the other slave. Call the amount of these two sums $p$; and the property which the slave leaves $a$.

Next, as to the residue of the slaves' property:

First. If the slave dies before the master, the master's son takes $\frac{1}{3} [a - p]$; the master's daughter $\frac{1}{4} [a - p]$, and the slave's daughter $\frac{1}{4} [a - p]$.

Second. If the slave dies after the master; the master's son is to receive $\frac{2}{3} p$, and the master's daughter $\frac{1}{3} p$; and then the master's son takes $\frac{1}{2} [a - p]$, and the slave's daughter $\frac{1}{2} [a - p]$. 
it between the son and the daughter (of the master), in such a manner, that the son receives twice as much as the daughter; and what then remains (from the heritage of the slave) is for the son alone, exclusive of the daughter; for the moiety of the heritage of the slave descends to the daughter of the slave, and the other moiety, according to the law of succession, to the son of the master, and there is nothing for the daughter (of the master).

It is the same, if a man on his death-bed emancipates a slave, besides whom he has no capital, and then the slave dies before his master.

If a man in his illness emancipates a slave, besides whom he possesses nothing, then that slave must ransom himself by two-thirds of his price. If the master has anticipated these two-thirds of his price and has spent them, then, upon the death of the master, the slave must pay two-thirds of what he retains.* But if the master has anticipated from him his whole price and spent it, then there is no claim against the slave, since he has already paid his entire price.

"Suppose that a man on his death-bed emancipates a slave, whose price is three hundred dirhems, not having any property besides; then the slave dies, leaving three hundred dirhems and a daughter." The

* The slave retains one-third of his price; and this he must redeem at two-thirds of its value; namely at \( \frac{2}{3} \times \frac{1}{3} = \frac{2}{9} \) of his original price.
computation is this:* Call the legacy to the slave thing. He has to return the remainder of his price, after the deduction of the legacy, or three hundred less thing. This ransom, of three hundred less thing, belongs to the master. Now the slave dies, and leaves thing and a daughter. She must receive the moiety of this, namely, one half of thing; and the master receives as much. Therefore the heirs of the master receive three hundred less half a thing, and this is twice as much as the legacy, which is thing, namely, two things. Reduce this by removing half a thing from the three hundred, and adding it to the two things. Then you have three hundred, equal to two things and a half. One thing is, therefore, as much as two-fifths of three hundred,

* Let the slave's original cost be $a$; the property which he dies possessed of, $\alpha$; what the master bequeaths to the slave, in emancipating him, $x$. Then the net property which the slave dies possessed of is $\alpha + x - a$. \( \frac{1}{3} [\alpha + x - a] \) belongs, by law, to the master; and \( \frac{1}{3} [\alpha + x - a] \) to the slave's daughter. The master's heirs, therefore, receive the ransom, $a - x$, and the inheritance, \( \frac{1}{3} [\alpha + x - a] \); that is, \( \frac{1}{3} [\alpha + a - x] \); and on the same principle as the slave, when emancipated, is allowed to ransom himself at two-thirds of his cost, the law of the case is that 2 are to be taken, where 1 is given.

\[ \therefore \frac{1}{2} [\alpha + a - x] = 2x \quad \therefore x = \frac{1}{3} [\alpha + a] \]

The daughter's share of the inheritance = \( \frac{1}{3} [3x - 2a] \)

The master's heirs receive.............. \( \frac{2}{3} [x + a] \)

If, as in the example, $\alpha = a$, $x = \frac{5}{3} a$; the daughter's share = \( \frac{1}{3} a \); the heirs of the master receive \( \frac{4}{3} a \).
namely, one hundred and twenty. This is the legacy (to the slave,) and the ransom is one hundred and eighty.  

"Some person on his sick-bed has emancipated a slave, whose price is three hundred dirhems; the slave then dies, leaving four hundred dirhems and ten dirhems of debt, and two daughters, and bequeathing to a person one-third of his capital; the master has twenty dirhems debts." The computation of this case is the following:* Call the legacy to the slave thing; his ransom is the remainder of his price, namely, three hundred less thing. But the slave, when dying, left four hundred dirhems; and out of this sum, his ransom, namely, three hundred less thing, is paid to the

* Let the slave's original cost = $a$; the property he dies possessed of = $a$; the debt he owes = $\epsilon$.

He leaves two daughters, and bequeaths to a stranger one-third of his capital.

The master owes debts to the amount $\mu$; where $a = 300$; $\alpha = 400$; $\epsilon = 10$; $\mu = 20$.

Let what the master gives to the slave, in emancipating him = $x$.

Slave's ransom = $a - x$; slave's property - slave's ransom = $\alpha + x - a$.

Slave's property - ransom - debt = $\alpha + x - a - \epsilon$.

Legacy to stranger = $\frac{1}{3} \left[ \alpha + x - a - \epsilon \right]$.

Residue = $\frac{2}{3} \left[ \alpha + x - a - \epsilon \right]$.

The master, and each daughter, are, by law, severally entitled to $\frac{1}{3} \times \frac{2}{3} \left[ \alpha + x - a - \epsilon \right]$.

The master's heirs receive altogether $a - x + \frac{5}{9} \left[ \alpha + x - a - \epsilon \right]$ or $\frac{3}{9} [a - x] + \frac{5}{9} [\alpha - \epsilon]$, which, on the principle that 2
master, so that one hundred dirhems and thing remain in the hands of the slave's heirs. Herefrom are (first) subtracted the debts, namely, ten dirhems; there remain then ninety dirhems and thing. Of this he has bequeathed one-third, that is, thirty dirhems and one-third of thing; so that there remain for the heirs sixty dirhems and two-thirds of thing. Of this the two daughters receive two-thirds, namely, forty dirhems and four-ninths of thing, and the master (104) receives twenty dirhems and two-ninths of thing, so that the heirs of the master obtain three hundred and twenty dirhems less seven-ninths of thing. Of this the debts of the master must be deducted, namely, twenty dirhems; there remain then three hundred dirhems less

are to be taken for I given, ought to be made equal to 2x.

But the author directs that the equation for determining x be

\[ \frac{1}{3} [a-x] + \frac{2}{3} [a-\varepsilon] - \mu = 2x \]

\[ \therefore x = \frac{1}{2} [7a + 2 (a-\varepsilon) - 9\mu] = 108 \]

Hence the slave receives, the debts which he owes, \( \varepsilon = 10 \)

+ the legacy to the stranger = \( \frac{1}{3} [9(a-\varepsilon) - 6a - 3\mu] = 66 \)

+ the inheritance of 1st daughter = \( \frac{1}{3} [6(a-\varepsilon) - 4a - 2\mu] = 44 \)

+ the inheritance of 2d daughter = \( \frac{1}{3} [6(a-\varepsilon) - 4a - 2\mu] = 44 \)

Total = \( \frac{1}{3} [21a + 4\varepsilon - 14a - 7\mu] = 164 \)

And the master takes \( \mu + 2x = \frac{1}{3} [4x - 4\varepsilon + 14a - 7\mu] = 236 \)

Had the slave died possessed of no property whatever, his ransom would have been 200.

His ransom, here stated, exclusive of the sum which the master inherits from him, or \( a-x, = 192 \).
seven-ninths of thing; and this sum is twice as much as the legacy of the slave, which was thing; or, it is equal to two things. Reduce this, by removing the seven-ninths of thing, and adding them to two things; there remain three hundred, equal to two things and seven-ninths. One thing is as much as nine twenty-fifths of eight hundred, which is one hundred and eight; and so much is the legacy to the slave.

If, on his sick-bed, he emancipates two slaves, besides whom he has no property, the price of each of them being three hundred dirhems; the master having anticipated and spent two-thirds of the price of one of them before he dies;* then only one-third of the price

* Were there the first slave only, who has paid off two-thirds of his original cost, the master having spent the money, that slave would have to complete his ransom by paying two-ninths of his original cost, that is 66\(\frac{2}{3}\) (see page 141).

Were there the second slave only, who has paid off none of his original cost, he would have to ransom himself at two-thirds of his cost; that is by paying 200 (see also page 141).

The master's heirs, in the case described in the text, are entitled to receive the same amount from the two slaves jointly, viz. 266\(\frac{2}{3}\), as they would be entitled to receive, according to the rule of page 141, from the two slaves, separately; but the payment of the sum is differently distributed; the slave who has paid two-thirds of his ransom being required to pay one-ninth only of his original cost; and the slave who has paid no ransom, being required to pay two-thirds of his own cost, and one-ninth of the cost of the first slave.
of this slave, who has already paid off a part of his ransom, belongs to the master; and thus the master's capital is the entire price of the one who has paid off nothing of his ransom, and one-third of the price of the other who has paid part of it; the latter is one hundred dirhems; the other three hundred dirhems: one-third of the amount, namely, one hundred and thirty-three dirhems and one third, is divided into two moieties among them; so that each of them receives sixty-six dirhems and two-thirds. The first slave, who has already paid two-thirds of his ransom, pays thirty-three dirhems and one-third; for sixty-six dirhems and two-thirds out of the hundred belong to himself as a legacy, and what remains of the hundred he must return. The second slave has to return two hundred and thirty-three dirhems and one-third.

"Suppose that a man, in his illness, emancipates two slaves, the price of one of them being three hundred dirhems, and that of the other five hundred dirhems; the one for three hundred dirhems dies, leaving a daughter; then the master dies, leaving a daughter likewise; and the slave leaves property to the amount of four hundred dirhems. With how much must every one ransom himself?"* The computation is this: Call

* Let A. be the first slave; his original cost \(a\); the property he dies possessed of \(a\); and let B. be the second slave; and his cost \(b\).
the legacy to the first slave, whose price is three hundred dirhems, thing. His ransom is three hundred dirhems less thing. The legacy to the second slave of a price of five hundred dirhems is one thing and two-thirds, and his ransom five hundred dirhems less one thing and two-thirds (viz. his price being one and two-thirds times the price of the first slave, whose ransom was thing, he must pay one thing and two-thirds for

Let \( x \) be that which the master gives to A. in emancipating him.

A.'s ransom is \( a - x \); and his property, minus his ransom, is \( a - a + x \).

A.'s daughter receives \( \frac{1}{2} [x - a + x] \), and the master's heirs receive \( \frac{1}{2} [a - a + x] \).

Hence the master receives altogether from A.,
\[
a - x + \frac{1}{2} [a - a + x] = \frac{1}{2} [a + a - x].
\]

B.'s ransom is \( b - \frac{b}{a} x \).

The master's heirs receive from A. and B. together \( \frac{1}{2} [a + a + 2b] - \frac{1}{2a} [a + 2b] x \); and this is to be made equal to twice the amount of the legacies to A. and B., that is,
\[
\frac{1}{2} [a + a + 2b] - \frac{1}{2a} [a + 2b] x = 2 - \frac{a + b}{a} x
\]
\[
\therefore \quad x = a \cdot \frac{a + a + 2b}{5a + 6b} = \frac{1700}{15} = 113\frac{1}{3}
\]
The master's heirs receive from A., \( \frac{2a[a + a + b] + 3ab}{5a + 6b} = 293\frac{1}{3} \).

A.'s daughter receives \( \frac{a + b}{5a + 6b} \) \( \frac{3a - 2a}{5a + 6} = 800 \times \frac{500}{3000} = 106\frac{2}{3} \).

The legacy to B. is \( b \cdot \frac{a + a + 2b}{5a + 6b} = 188\frac{8}{9} \); his ransom is
\[
b \cdot \frac{a + a + 2b}{5a + 6b} = 311\frac{1}{3}
\]
The master's heirs receive from A. and B. together
\[
2 \cdot \frac{a + b}{5a + 6b} \cdot \frac{a + a + 2b}{5a + 6b} = 604\frac{4}{9}.
\]
his ransom). Now the slave for three hundred dirhems dies, and leaves four hundred dirhems. Out of this his ransom is paid, namely, three hundred dirhems less thing; and in the hands of his heirs remain one hundred dirhems plus thing: his daughter receives the moiety of this, namely, fifty dirhems and half a thing; and what remains belongs to the heirs of the master, namely, fifty dirhems and half a thing. This is added to the three hundred less thing; the sum is three hundred and fifty less half a thing. Add thereto the ransom of the other, which is five hundred dirhems less one thing and two-thirds; thus, the heirs (106) of the master have obtained eight hundred and fifty dirhems less two things and one-sixth; and this is twice as much as the two legacies together, which were two things and two-thirds. Reduce this, and you have eight hundred and fifty dirhems, equal to seven things and a half. Make the equation; one thing will be equal to one hundred and thirteen dirhems and one-third. This is the legacy to the slave, whose price is three hundred dirhems. The legacy to the other slave is one and two-thirds times as much, namely, one hundred and eighty-eight dirhems and eight-ninths, and his ransom three hundred and eleven dirhems and one-ninth.

"Suppose that a man in his illness emancipates two slaves, the price of each of whom is three hundred dirhems; then one of them dies, leaving five hundred dirhems and a daughter; the master having left a son."
Computation:* Call the legacy to each of them thing; the ransom of each will be three hundred less thing; then take the inheritance of the deceased slave, which is five hundred dirhems, and subtract his ransom, which is three hundred less thing; the remainder of his inheritance will be two hundred plus thing. Of this, one hundred dirhems and half a thing return to the master by the law of succession, so that now altogether four hundred dirhems less a half thing are in the hands of the master's heirs. Take also the ransom of the other slave, namely, three hundred dirhems less thing; then the heirs of the master obtain seven hundred dir-

* The first slave is A.; his cost a; his property \( \alpha \); he leaves a daughter.

The second slave is B.; his cost b.

Then (as in page 147) \( \frac{\alpha - a + x}{2} \) goes to the daughter; and \( x = a \frac{e + a + 2b}{5a + 6b} \)

The daughter receives \( [a + b] \frac{3a - 2a}{5a + 6b} \)

The master receives from A. \( \frac{2a[a + a + b] + 3ab}{5a + 6b} \)

and the master receives from A. and B. together

\[ \frac{2[a + b]}{5a + 6b} \]

But if \( b = a \ldots \quad x = \frac{1}{11} [x + 3a] = 127\frac{3}{11} \]

The daughter receives \( \quad \frac{2}{11} [3a - 2a] = 163\frac{3}{11} \]

The master receives from A. \( \quad \frac{1}{11} [5a + 4a] = 336\frac{4}{11} \)

The master receives from B. \( \quad \frac{1}{11} [8a - a] = 172\frac{9}{11} \)

The master receives from A and B. \( \quad \frac{4}{11} [a + 3a] = 509\frac{9}{11} \)

If \( b = 0, \)

The daughter receives \( \frac{1}{3} [3a - 2a] \)

The master \( \ldots \quad \frac{2}{3} [x + a] \), as in page 142.
hems less one thing and a half, and this is twice as much as the sum of the two legacies of both, namely (107) two things, consequently as much as four things. Remove from this the one thing and a half: you find seven hundred dirhems, equal to five things and a half.

Make the equation. One thing will be one hundred and twenty-seven dirhems and three-elevenths.

"Suppose that a man in his illness emancipate a slave, whose price is three hundred dirhems, but who has already paid off to his master two hundred dirhems, which the latter has spent; then the slave dies before the death of the master, leaving a daughter and three hundred dirhems."

Computation: Take the property left by the slave, namely, the three hundred, and add thereto the two hundred, which the master has spent; this together makes five hundred dirhems. Subtract from this the ransom, which is three hundred less thing

* The slave A. dies before his master, and leaves a daughter. His cost is \( a \), of which he has redeemed \( \hat{a} \), which the master has spent; and he leaves property \( x \).

Then the daughter receives \( \frac{1}{2} [x + \hat{a} - a + x] \)

The master receives altogether \( \frac{1}{3} [x + \hat{a} + a - x] \)

The master's heirs receive.... \( \frac{1}{2} [x - \hat{a} + a - x] \)

And \( \frac{1}{2} [x - \hat{a} + a - x] = 2x \quad \therefore x = \frac{1}{2} [x - \hat{a} + a] \)

Hence the daughter receives \( \frac{1}{2} [3x + 2\hat{a} - 2a] = 140 \)

The master's heirs ......... \( \frac{1}{2} [2x - 2\hat{a} + 2a] = 160 \)

The master receives, in toto, \( \frac{1}{3} [2x + 3\hat{a} + 2a] = 360 \)

If the slave had not advanced, or the master had not spent \( \hat{a} \), the daughter would have received \( \frac{1}{3} [3x + 3\hat{a} - 2a] = 180 \)

and the master would have received \( \frac{1}{3} [2x + 2\hat{a} + 2a] = 320. \)
(151)

(since his legacy is thing); there remain two hundred dirhems plus thing. The daughter receives the moiety of this, namely, one hundred dirhems plus half a thing; the other moiety, according to the laws of inheritance, returns to the heirs of the master, being likewise one hundred dirhems and half a thing. Of the three hundred dirhems less thing there remain only one hundred dirhems less thing for the heirs of the master, since two hundred are spent already. After the deduction of these two hundred which are spent, there remain with the heirs two hundred dirhems less half thing, and this is equal to the legacy of the slave taken twice; or the moiety of it, one hundred less one-fourth of thing, is equal to the legacy of the slave, which is thing. Remove from this the one-fourth of thing; then you have one hundred dirhems, equal to one thing and one-fourth. One thing is four-fifths of it, namely, eighty dirhems. This is the legacy; and the ransom is two hundred and twenty dirhems. Add the inheritance of the slave, which is three hundred, to two hundred, which (108) are spent by the master. The sum is five hundred dirhems. The master has received the ransom of two hundred and twenty dirhems; and the moiety of the remaining two hundred and eighty, namely, one hundred and forty, is for the daughter. Take these from the inheritance of the slave, which is three hundred; there remain for the heirs one hundred and sixty dirhems, and this is twice as much as the legacy of the slave, which was thing.
"Suppose that a man in his illness emancipates a slave, whose price is three hundred dirhems, but who has already advanced to the master five hundred dirhems; then the slave dies before the death of his master, and leaves one thousand dirhems and a daughter. The master has two hundred dirhems debts."* Computation: Take the inheritance of the slave, which is one thousand dirhems, and the five hundred, which the master has spent. The ransom from this is three hundred less thing. There remain therefore twelve hundred plus thing. The moiety of this belongs to the daughter: it is six hundred dirhems plus half a thing. Subtract it from the property left by the slave, which was one

* A.'s price is \( a \); he has advanced to his master \( d \); he leaves property \( a \). He dies before his master, and leaves a daughter.

The master's debts are \( \mu \); \( x \) is what \( \Lambda \) receives, in being emancipated; \( a-x \) is the ransom; \( \frac{1}{2} [x+d-a+x] \) is what the daughter receives.

Then \( a-\frac{1}{2} [x+d-a+x] \) is what remains to the master; and \( a-\frac{1}{2} [x+d-a+x]-\mu \) is what remains to him, after paying his debts; and this is to be made equal to \( 2x \).

Whence \( x=\frac{1}{3} [\alpha +\alpha -\alpha -2\mu] \)

Hence the daughter receives \( \ldots \ldots \ldots \frac{1}{3} [3\alpha-2\alpha+2\alpha-\mu]=640 \)
The mother receives, inclusive of the debt \( \ldots \ldots \ldots \frac{1}{3} [2\alpha+2\alpha-2\alpha+\mu]=360 \)
The master receives, exclusive of the debt \( \ldots \ldots \ldots \frac{1}{3} [2\alpha+2\alpha-2\alpha-4\mu]=160 \)

If the mode given in page 142 had been followed, it would have given \( x=\frac{1}{3} [\alpha +\alpha +\alpha -2\mu] \) and the daughter's portion \( =\frac{1}{3} [3\alpha-2\alpha+3\alpha-\mu]=740 \).
thousand dirhems: there remain four hundred dirhems, less half thing. Subtract herefrom the debts of the master, namely, two hundred dirhems; there remain two hundred dirhems less half thing, which are equal to the legacy taken twice, which is thing; or equal to two things. Reduce this, by means of the half thing. Then you have two hundred dirhems, equal to two things and a half. Make the equation. You find one thing, equal to eighty dirhems; this is the legacy. Add now the property left by the slave to the sum which he has advanced to the master: this is fifteen hundred dirhems. Subtract the ransom, which is two hundred and twenty dirhems; there remain twelve hundred and eighty dirhems, of which the daughter receives the moiety, namely, six hundred and forty dirhems. Subtract this from the inheritance of the slave, which is one thousand dirhems: there remain three hundred and sixty dirhems. Subtract from this the debts of the master, namely, two hundred dirhems; there remain then one hundred and sixty dirhems for the heirs of the master, and this is twice as much as the legacy of the slave, which was thing.

"Suppose that a man on his sick-bed emancipates a slave, whose price is five hundred dirhems, but who has already paid off to him six hundred dirhems. The master has spent this sum, and has moreover three hundred dirhems of debts. Now the slave dies, leaving his mother and his master, and property to the amount of seventeen hundred and fifty dirhems, with two hundred
dirhems debts." Computation:* Take the property left by the slave, namely, seventeen hundred and fifty dirhems, and add to it what he has advanced to the master, namely, six hundred dirhems; the sum is two thousand three hundred and fifty dirhems. Subtract from this the debts, which are two hundred dirhems, and the ransom, which is five hundred dirhems less thing, since the legacy is thing; there remain then sixteen hundred and fifty dirhems plus thing. The mother receives herefrom one-third, namely, five hundred and fifty dirhems less one-third of thing. Subtract now this and the debts, which are two hundred dirhems, from the actual inheritance of the slave, which is seventeen hundred and fifty; there remain one thousand dirhems less one-third of thing. Subtract from this the debts of the master, namely, three hundred

* A. dies before his master, and leaves a mother. His price was \( a \); he has redeemed \( \dot{a} \), which the master has spent. The property he leaves is \( a \). He owes debts \( \epsilon \). The master owes debts \( \mu \).

\[
\frac{1}{3} \left[ x + \dot{a} - a + x - \epsilon \right] \text{ is the mother's.}
\]

\[
a - \frac{1}{3} \left[ x + \dot{a} - a + x - \epsilon \right] - \epsilon - \mu = 2x = \text{the master's, after paying his debts.}
\]

Hence \[ x = \frac{1}{3} \left[ 2a + a - \dot{a} - 2\epsilon - 3\mu \right] = 300 \]

Mother's \[ = \frac{1}{3} \left[ 3a - 2a + 2\dot{a} - 3\epsilon - \mu \right] = 650 \]

Master's, without \( \mu \) \[ = \frac{1}{3} \left[ 4a + 2a - 2\dot{a} - 4\epsilon - 6\mu \right] = 600 \]

Mother's, with \( \mu \) \[ = \frac{1}{3} \left[ 4a + 2a - 2\dot{a} - 4\epsilon + \mu \right] = 900 \]

A. receives, inclusive of \( \epsilon \) \[ = \frac{1}{3} \left[ 3a - 2a + 2\dot{a} + 4\epsilon - \mu \right] = 850 \].
dirhems; there remain seven hundred dirhems less one-third of thing. This is twice as much as the legacy of the slave, which is thing. Take the moiety: then three hundred and fifty less one-sixth of thing are equal to one thing. Reduce this, by means of the one-sixth of thing; then you have three hundred and fifty, equal to one thing and one-sixth. One thing will then be equal to six-sevenths of the three hundred and fifty, namely, three hundred dirhems; this is the legacy. Add now the property left by the slave to what the master has spent already; the sum is two thousand three hundred and fifty dirhems. Subtract herefrom the debts, namely, two hundred dirhems, and subtract also the ransom, which is as much as the price of the slave less the legacy, that is, two hundred dirhems; there remain nineteen hundred and fifty dirhems. The mother receives one-third of this, namely, six hundred and fifty dirhems. Subtract this and the debts, which are two hundred dirhems, from the property actually left by the slave, which was seventeen hundred and fifty dirhems; there remain nine hundred dirhems. Subtract from this the debts of the master, which are three hundred dirhems; there remain six hundred dirhems, which is twice as much as the legacy.

"Suppose that some one in his illness emancipates a slave, whose price is three hundred dirhems: then the slave dies, leaving a daughter and three hundred dirhems; then the daughter dies, leaving her husband and
three hundred dirhems; then the master dies.” Computation:* Take the property left by the slave, which is three hundred dirhems, and subtract the ransom, which (111) is three hundred less thing; there remains thing, one half of which belongs to the daughter, while the other half returns to the master. Add the portion of the daughter, which is half one thing, to her inheritance, which is three hundred; the sum is three hundred dirhems plus half a thing. The husband receives the moiety of this; the other moiety returns to the master, namely one hundred and fifty dirhems plus one-fourth of thing. All that the master has received is therefore four hundred and fifty less one-fourth of thing; and this is twice as much as the legacy; or the moiety of it is as much as

* A. is emancipated by his master, and then dies, leaving a daughter, who dies, leaving a husband. Then the master dies.

A.’s price = $a$; his property $a$. What he receives from the master $= x$.

The daughter’s property = $b$

A.’s ransom $= a - x$. The daughter inherits $\frac{1}{2}[a - a + x]$, and $\frac{1}{2}[a - a + x]$ goes to the master.

$\frac{1}{2}[b + \frac{1}{2}[a - a + x]]$ goes to the daughter’s husband

and $\frac{1}{2}[b + \frac{1}{2}[a - a + x]]$ goes to the master.

Hence, according to the author, we are to make

$a - x + \frac{1}{2}[a - a + x] + \frac{1}{2}[b + \frac{1}{2}[a - a + x]] = 2x$

$\therefore x = \frac{1}{2}[3a + a + 2b] = 200$

Daughter’s share $= \frac{1}{2}[6a - 4a + b] = 100$

Husband’s $= \frac{1}{2}[3a - 2a + 5b] = 200$

Master’s $= \frac{1}{2}[2a + 6a + 4b] = 400$. 
the legacy itself, namely, two hundred and twenty-five dirhems less one-eighth thing are equal to thing. Reduce this by means of one-eighth of thing, which you add to thing; then you have two hundred and twenty-five dirhems, equal to one thing and one-eighth. Make the equation: one thing is as much as eight-ninths of two hundred and twenty-five, namely, two hundred dirhems.

"Suppose that some one in his illness emancipates a slave, of the price of three hundred dirhems; the slave dies, leaving five hundred dirhems and a daughter, and bequeathing one-third of his property; then the daughter dies, leaving her mother, and bequeathing one-third of her property, and leaving three hundred dirhems." Computation:* Subtract from the property left

* A. is emancipated, and dies, leaving a daughter, and bequeathing one-third of his property to a stranger. The daughter dies, leaving a mother, and bequeathing one-third of her property to a stranger.

A.'s price is \(\alpha\); his property is \(\alpha\).

The daughter's property is \(\frac{\alpha}{3}\).

A.'s ransom is \(\alpha-x\); \(\alpha-x+x\) is his property, clear of ransom.

\(\frac{1}{3}[\alpha-x+x]\) goes to the stranger; and the like amount to A.'s daughter, and to the master.

\(\frac{1}{3}[3\alpha-x+\alpha-x]\) is the property left by the daughter.

\(\frac{1}{3}[3\alpha-x+\alpha-x]\) is the bequest of the daughter to a stranger.

\(\frac{1}{9}[3\alpha-x+\alpha-x]\) is the residue, of which \(\frac{1}{9}d\),

viz. \(\frac{1}{9}d\) \(\frac{1}{3}[3\alpha-x+\alpha-x]\) is the mother's,

and \(\frac{1}{9}d\) \(\frac{1}{3}[3\alpha-x+\alpha-x]\) is the master's;
by the slave his ransom, which is three hundred dirhems less thing; there remain two hundred dirhems plus thing. He has bequeathed one-third of his property, that is, sixty-six dirhems and two-thirds plus one-third of thing. According to the law of succession, sixty-six dirhems and two-thirds and one-third of thing belong to the master, and as much to the daughter. Add this to the property left by her, which is three hundred dirhems: the sum is three hundred and sixty-six dirhems and two-thirds and one-third of thing. She has bequeathed one-third of her property, that is, one hundred and twenty-two dirhems and two-ninths and one-ninth of thing; and there remain two hundred and forty-four dirhems and four-ninths and two-ninths of thing. The mother receives one-third of this, namely, eighty-one dirhems and four-ninths and one-third of one-ninth of a dirhem plus two-thirds of one-ninth of thing. The remainder returns to the master; it is a hundred and sixty-two dirhems and eight-ninths and two-thirds of one-ninth of a dirhem plus one-ninth and one-third of one-ninth of thing, as his share of the heritage.

Hence, according to the author, we are to make

\[ a - x + \frac{3}{a} \left[ a - a + x \right] + \frac{4}{3a} \left[ 3a + a - a + x \right] = 2x \]

Therefore \[ x = \frac{1}{6} \left[ 13a + 14a + 12a \right] = 210 \frac{6}{17} \]

The daughter's share \[ = \frac{1}{6} \left[ 27a - 18a + 4a \right] = 136 \frac{1}{4} \]

The daughter's bequest \[ = \frac{1}{6} \left[ 9a - 6a + 24a \right] = 145 \frac{1}{4} \]

The mother's share \[ = \frac{2}{6} \left[ 3a - 2a + 8a \right] = 97 \frac{1}{7} \]

The master's \[ = \frac{3}{6} \left[ 13a + 14a + 12a \right] = 420 \frac{1}{4} \].
Thus the master's heirs have obtained five hundred and twenty-nine dirhems and seventeen twenty-sevenths of a dirhem less four-ninths and one-third of one-ninth of thing; and this is twice as much as the legacy, which is thing. Halve it: You have two hundred and sixty-four dirhems and twenty-two twenty-sevenths of a dirhem, less seven twenty-sevenths of thing. Reduce it by (113) means of the seven twenty-sevenths which you add to the one thing. This gives one hundred and sixty-four dirhems and twenty-two twenty-sevenths, equal to one thing and seven twenty-sevenths of thing. Make the equation, and adjust it to one single thing, by subtracting from it as much as seven thirty-fourths of the same. Then one thing is equal to two hundred and ten dirhems and five-seventeenths; and this is the legacy.

"Suppose that a man in his illness emancipates a slave, whose price is one hundred dirhems, and makes to some one a present of a slave-girl, whose price is five hundred dirhems, her dowry being one hundred dirhems, and the receiver cohabits with her." Abu Hanifah says: The emancipation is the more important act, and must first be attended to.

Computation:* Take the price of the girl, which is

* The price of the slave-girl being $a$; and what she receives on being emancipated $x$, her ransom is $a - x$.
If her dowry is $e$, he that receives her, takes $a + x$. 
five hundred dirhems; and remember that the price of the slave is one hundred dirhems. Call the legacy of the donee thing. The emancipation of the slave, whose price is one hundred dirhems, has already taken place. He has bequeathed one thing to the donee. Add the dowry, which is one hundred dirhems less one-fifth thing. Then in the hands of the heirs are six hundred dirhems less one thing and one-fifth of thing. This is twice as much as one hundred dirhems and thing; the moiety of it is equal to the legacy of the two, namely, three hundred less three-fifths of thing. Reduce this by removing the three-fifths of thing from three hundred, and add the same to one thing. This gives three hundred dirhems, equal to one thing and three-fifths and one hundred dirhems. Subtract now from three hun-

Hence, according to the author, we are to make

$$a-x=2\left[\alpha+x\right]; \text{ whence } x=\frac{a-2x}{3}$$

And her ransom is $\frac{3}{2}[\alpha+x]$

But if a male slave be at the same time emancipated by the master, the donee must pay the ransom of that slave. If his price was $b$, $b-\frac{b}{a}x$ is his ransom.

Hence, according to the author, we are to make the sum of the two ransoms, viz. $a-x+b-\frac{b}{a}x=2[\alpha+x]$

$$\therefore \ a+b-2x=[3+\frac{b}{a}]x \ \ \therefore \ x=a \ \frac{a+b-2x}{3a+b}=125$$

The donee pays ransom, in respect of the slave-girl $(a-x)=375$ and he pays ransom for the male slave $\ldots \ldots \ldots \ b-\frac{b}{a}x=75$. 
dred the one hundred, on account of the other one hundred. There remain two hundred dirhems, equal to one thing and three-fifths. Make the equation with this. One thing will be five-eighths of what you have; (114) take therefore five-eighths of two hundred. It is one hundred and twenty-five. This is thing; it is the legacy to the person to whom he had presented the girl.

"Suppose that a man emancipates a slave of a price of one hundred dirhems, and makes to some person a present of a slave girl of the price of five hundred dirhems, her dowry being one hundred dirhems; the donee cohabits with her, and the donor bequeaths to some other person one-third of his property." According to the decision of Abu Hanifah, no more than one-third can be taken from the first owner of the slave-girl; and this one-third is to be divided into two equal parts between the legatee and the donee. Computation:* Take the price of the girl, which is five hundred dirhems. The legacy out of this is thing; so that the heirs obtain five hundred dirhems less thing; and the dowry is one hundred less one-fifth of thing; consequently they

* The same notation being used as in the last example, the equation for determining \( x \), according to the author, is to be

\[
a - x + b - \frac{b}{a} x - x = 2 \left[ x + 2x \right]
\]

\[
\therefore x = \frac{a}{6a + b} \left[ a + b - 2x \right] = 64\frac{3}{4}.
\]
obtain six hundred dirhems less one thing and one-fifth of thing. He bequeaths to some person one third of his capital, which is as much as the legacy of the person who has received the girl, namely, thing. Consequently there remain for the heirs six hundred less two things and one-fifth, and this is twice as much as both their legacies taken together, namely, the price of the slave plus the two things bequeathed as legacies. Halve it, and it will by itself be equal to these legacies: it is then three hundred less one and one-tenth of thing. Reduce this by means of the one and one-tenth of thing. Then you have three hundred, equal to three things and one-tenth, plus one hundred dirhems. Remove one hundred on account of (the opposite) one hundred; there remain two hundred, equal to three things and one-tenth. Make now the reduction. One thing will be as much as thirty-one parts of the sum of dirhems which you have; and just so much will be the legacy out of the two hundred; it is sixty-four dirhems and sixteen thirty-one parts.

"Suppose that some one emancipates a slave girl of the price of one hundred dirhems, and makes to some person a present of a slave girl, which is five hundred dirhems worth; the receiver cohabits with her, and her dowry is one hundred dirhems; the donor bequeaths to some other person as much as one-fourth of his capital." Abu Hanifah says: The master of the girl cannot be required to give up more than one-third, and the legatee, who is to receive one-fourth, must give up
one-fourth. Computation: The price of the girl is five hundred dirhems. The legacy out of this is thing; there remain five hundred dirhems less thing. The dowry is one hundred dirhems less one-fifth of thing; thus the heirs obtain six hundred dirhems less one and one-fifth of thing. Subtract now the legacy of the person to whom one-fourth has been bequeathed, namely, three-fourths of thing; for if one-third is thing then one-fourth is as much as three-fourths of the same.

There remain then six hundred dirhems less one thing and thirty-eight fortieths. This is equal to the legacy taken twice. The moiety of it is equal to the legacies by themselves, namely, three hundred dirhems less thirty-nine fortieths of thing. Reduce this by means of the latter fraction. Then you have three hundred dirhems, equal to one hundred dirhems and two things and twenty-nine fortieths. Remove one hundred on account of the other one hundred. There remain two hundred dirhems, equal to two things and twenty-nine fortieths. Make the equation. You will then find one thing to be equal to seventy-three dirhems and forty-three one-hundred-and-ninths dirhems.

* The same notation being used as in the two former examples, the equation for determining \( x \), according to the author, is

\[
\frac{a-x+b}{a} - \frac{b}{a} \cdot x - \frac{3}{4} x = 2 [x + 1 \frac{3}{4} x]
\]

Whence

\[
\frac{4a}{21a+4b} [a + b - 2x] = 7 \frac{1}{3} \frac{1}{10} \frac{5}{4}.
\]
On return of the Dowry.

"A man, in the illness before his death, makes to some one a present of a slave girl, besides whom he has no property. Then he dies. The slave girl is worth three hundred dirhems, and her dowry is one hundred dirhems. The man to whom she has been presented, cohabits with her." Computation:* Call the legacy of the person to whom the girl is presented, thing. Subtract this from the donation: there remain three hundred less thing. One-third of this difference returns to the donor on account of dowry (since the dowry is one-third of the price): this is one hundred dirhems less one-third of thing. The donor's heirs obtain, therefore, four hundred less one and one-third of thing, which is equal to twice the legacy, which is thing, or to two things. Transpose the one and one-third thing from the four hundred, and add it to the two things; then you have four hundred, equal to three things and one-third. One thing is, therefore, equal to three-tenths of it, or to one hundred and twenty dirhems, and this is the legacy.

* Let \( a \) be the slave-girl's price — \( x \) her dowry.

Then, according to the author, we are to make

\[
a - x + x - \frac{a}{a} x = 2x
\]

Therefore

\[
\frac{a}{3a + x} \left[a + x\right] = \frac{3}{10} \times 400 = 120
\]

The donee is to receive the girl's dowry, worth 400, for 280.
"Or, suppose that he, in his illness, has made a present of the slave girl, her price being three hundred, her dowry one hundred dirhems; and the donor dies, after having cohabited with her." Computation: * Call the legacy thing: the remainder is three hundred less thing. The donor having cohabited with her, the dowry remains with him, which is one-third of the legacy, since the dowry is one-third of the price, or one-third of thing. Thus the donor's heirs obtain three (117) hundred less one and one-third of thing, and this is twice as much as the legacy, which is thing, or equal to two things. Remove the one and one-third of thing, and add the same to the two things. Then you have three hundred, equal to three things and one-third. One thing is, therefore, three-tenths of it, namely ninety dirhems. This is the legacy.

If the case be the same, and both the donor and donee have cohabited with her; then the Computation

* If the donor has cohabited with the slave-girl, the donor's heirs are to retain the dowry, but must allow the donee, in addition to the legacy $x$, the further sum of $\frac{a}{a}x$;

The ransom is then $a-x-\frac{a}{a}x$, which according to the author is to be made equal to $2x$.

$$\text{Whence } x = \frac{a^2}{3x+a} = 90$$

The donee is to receive the girl, worth 300, for 210.
is this:* Call the legacy thing; the deduction is three hundred dirhems less thing. The donor has ceded the dowry to the donee by (the donee's) having cohabited with her: this amounts to one-third of thing: and the donee cedes one-third of the deduction, which is one hundred less one-third of thing. Thus, the donor's heirs obtain four hundred less one and two-thirds of thing, which is twice as much as the legacy. Reduce this, by separating the one and two-thirds of thing from four hundred, and add them to the two things. Then you have four hundred things, equal to three things and two-thirds. One thing of these is three-elevenths of four hundred; namely, one hundred and

* If the donor has previously cohabited with the slave-girl, it appears from the last example, that the donee is entitled to ransom her for \( a - x - \frac{a}{a} x \).

If the donee cohabits with the slave-girl, it appears from the last example but one, that he is entitled to redeem the dowry, \( x \), for \( a - \frac{a}{a} x \)

The redemption of the girl and dowry is

\[ a - x - \frac{a}{a} x + a - \frac{a}{a} x, \]

which, according to the author, is to be made equal to \( 2x \).

That is \( a + x - \frac{a + 2a}{a} x = 2x \)

Whence \( x = \frac{a}{3a + 2a} \times [a + x] = 109^{11} \)

The donee is to receive the girl and dowry, worth 400, for \( 290^{11} \).
nine dirhems and one-eleventh. This is the legacy. The deduction is one hundred and ninety dirhems and ten-elevenths. According to Abu Hanifah, you call the thing a legacy, and what is obtained on account of the dowry is likewise a legacy.

If the case be the same, but that the donor, having cohabited with her, has bequeathed one-third of his (118) capital, then Abu Hanifah says, that the one-third is halved between the donee and the legatee. Computation: * Call the legacy of the person to whom the slave-girl has been given, thing. After the deduction of it, there remain three hundred, less thing. Then take the dowry, which is one-third of thing; so that the donor retains three hundred less one and one-third of thing; the donee's legacy being, according to Abu Hanifah, one and one-third of thing; according to other lawyers, only thing. The legatee, to whom one-third is bequeathed, receives as much as the legacy of the donee, namely, one and one-third of thing. The donor thus retains three hundred, less two things and

* The second case is here solved in a different way.

\[ a - x - \frac{a}{a}x = 2 \left[ x + \frac{a}{a}x \right] \]

\[ \therefore \ x = \frac{a^2}{3[a+a]} \]

This being halved between the legatee and donee becomes

\[ \frac{a^2}{6[a+a]} = 37\frac{1}{2} \]

The donee receives the girl, worth 300, for 262\(\frac{3}{2}\).
two-thirds—equal to twice the two legacies, which are two things and two-thirds. The moiety of this, namely, one hundred and fifty less one and one-third of thing, must, therefore, be equal to the two legacies. Reduce it, by removing one and one-third of thing, and adding the same to the two legacies (things). Then you find one hundred and fifty, equal to four things. One thing is one-fourth of this, namely, thirty-seven and a half.

If the case be, that both the receiver and the donor have cohabited with her, and the latter has disposed of one-third of his capital by way of legacy; then the computation,* according to Abu Hanifah, is, that you call the legacy thing. After the deduction of it, there remain three hundred less thing. Then the dowry is taken, which is one hundred less one-third of thing; so that there are four hundred dirhems less one and one-third of thing. The sum returned from the dowry is one-third of thing; and the legatee, who is to receive one-third, obtains as much as the legacy of the first, namely, thing and one-third of thing. Thus there

* According to the author's rule, which is purely arbitrary,

\[ a - 2x + \frac{a}{a} - \frac{3a}{a} x = 4 \left[ 1 + \frac{a}{a} \right] x \]

Whence \( x = a \frac{a + a}{6a + 7a} = 48 \)

The donee will have to redeem the girl and dowry, worth 400, for 352.
remain four hundred dirhems less three things, equal to twice the legacy, namely, two things and two-thirds. (119) Reduce this, by means of the three things, and you find four hundred, equal to eight things and one-third. Make the equation with this: one thing will be forty-eight dirhems.

“Suppose that a man on his sick-bed makes to another a present of a slave-girl, worth three hundred dirhems, her dowry being one hundred dirhems; the donee cohabits with her, and afterwards, being also on his sick-bed, makes a present of her to the donor, and the latter cohabits with her. How much does he acquire by her, and how much is deducted?”

* We have here the only instance in the treatise of a simple equation, involving two unknown quantities. For what the donee receives is one unknown quantity; and what the donor receives back again from the donee, called by the author “part of thing,” is the other unknown quantity.

Let what the donee receives = $x$, and what the donor receives = $y$.

Then, retaining the same notation as before, according to the author, the donee receives, on the whole

$$x - y - \left[ a - \frac{a}{x} \right] + \frac{a}{x} [x - y] = 2y$$

and the donor receives, on the whole

$$a - x + y + \left[ a - \frac{a}{x} \right] - \frac{a}{x} [x - y] = 2 \left[ x + \frac{a}{x} [x - y] \right]$$

Whence $x = \frac{1}{2} \frac{a}{4a^2 + 5ax - a^2} \left[ 3a^2 + 3ax - 2a^2 \right] = 102$

$$y = \frac{1}{2} \frac{a}{4a^2 + 5ax - a^2} \left[ a^2 - 2ax^2 \right] = 21$$

But
putation: Take the price, which is three hundred dirhems; the legacy from this is thing; there remain with the donor's heirs three hundred less thing; and the donee obtains thing. Now the donee gives to the donor part of thing: consequently, there remains only thing less part of thing for the donee. He returns to the donor one hundred less one-third of thing; but takes the dowry, which is one-third of thing, less one-third of part of thing. Thus he obtains one and two-thirds thing less one hundred dirhems and less one and one-third of part of thing. This is twice as much as part of thing; and the moiety of it is as much as part of thing, namely, five-sixths of thing less fifty dirhems and less two-thirds of part of thing. Reduce this by removing two-thirds of part of thing and fifty dirhems. Then you have five-sixths of thing, equal to one and two-thirds of part of thing plus fifty dirhems. Reduce this to one single part of thing, in order to know what the amount of it is. You effect this by taking three-fifths of what you have. Then one part of thing plus thirty dirhems is equal to half a thing; and one-half thing less thirty dirhems is equal to part of thing, which is the legacy returning from the donee to the donor. Keep this in memory.

Then return to what has remained with the donor;

But the reasons for reducing the question to these two equations are not given by the author, and seem to depend on the dicta of the sages of the Arabian law.
this was three hundred less thing: hereto is now added the part of thing, or one-half thing less thirty dirhems. Thus he obtains two hundred and seventy less half one thing. He further takes the dowry, which is one hundred dirhems less one-third thing, but has to return a dowry, which is one-third of what remains of thing after the subtraction of part of thing, namely, one-sixth of thing and ten dirhems. Thus he retains three hundred and sixty less thing, which is twice as much as thing and the dowry, which he has returned. Halve it: then one hundred and eighty less one-half thing are equal to thing and that dowry. Reduce this, by removing one-half thing and adding it to the thing and the dowry: you find one hundred and eighty dirhems, equal to one thing and a half plus the dowry which he has returned, and which is one-sixth thing and ten dirhems. Remove these ten dirhems; there remain one hundred and seventy dirhems, equal to one and two-thirds things. Reduce this, in order to ascertain what the amount of one thing is, by taking three-fifths of what you have; you find that one hundred and two are equal to thing, which is the legacy from the donor to the donee: and the legacy from the donee to the donor is the moiety of this, less thirty dirhems, namely, twenty-one.
(121) "Suppose that a man, on his sick-bed, deliver to some one thirty dirhems in a measure of victuals, worth ten dirhems; he afterwards dies in his illness; then the receiver returns the measure and returns besides ten dirhems to the heirs of the deceased." Computation: He returns the measure, the value of which is ten dirhems, and places to the account of the deceased twenty dirhems; and the legacy out of the sum so placed is thing; thus the heirs obtain twenty less thing, and the measure. All this together is thirty dirhems less thing, equal to two things, or equal to twice the legacy. Reduce it by separating the thing from the thirty, and adding it to the two things. Then, thirty are equal to three things. Consequently, one thing must be one-third of it, namely, ten, and this is the sum which he obtains out of what he places to the account of the deceased.

"Suppose that some one on his sick-bed delivers to a person twenty dirhems in a measure worth fifty dirhems; he then repeals it while still on his sick bed, and dies after this. The receiver must, in this case, return four-ninths of the measure, and eleven dirhems and one-ninth."* Computation: You know that the

* Let $a$ be the gift of money; and the value of the measure $m \times a$.

It appears from the context that the donee is to pay the heirs $\frac{2}{3}ma$. 
price of the measure is two and a half times as much as the sum which the donor has given the donee in money; and whenever the donee returns anything from the money capital, he returns from the measure as much as two and a half times that amount. Take now from the measure as much as corresponds to one thing, that is, two things and a half, and add this to what remains from the twenty, namely, twenty less thing. Thus the heirs of the deceased obtain twenty dirhems and one thing and a half. The moiety of this is the legacy, namely, ten dirhems and three-fourths of thing; and this is one-third of the capital, namely, sixteen dirhems and two-thirds. Remove now ten dirhems on account of the opposite ten; there remain six dirhems and two-thirds, equal to three-fourths of thing. Complete the thing, by adding to it as much as one-third of the same; and add to the six dirhems and two-thirds

It is arbitrary how he shall apportion this sum between the money capital and the measure.

If he pays on the money capital $p\cdot a$
and on the measure $\ldots \ldots \ldots \ldots q\cdot ma$
we have the equation $p\cdot a + q\cdot ma = \frac{2}{3} ma$

or $p + q\cdot m = \frac{2}{3} m$

The author assumes $p = \frac{m}{2} \cdot q$

Whence $q = \frac{2}{3}$, and $p = \frac{5}{6}$, and therefore the donee pays on the money capital $\ldots \frac{5}{6} a = 11\frac{1}{2}$
and on the measure $\ldots \frac{2}{3} ma = 22\frac{2}{5}$

Total $\ldots \ldots \ldots \ldots 33\frac{1}{3}$. 
likewise one-third of the same, namely, two dirhems and two-ninths; this yields eight dirhems and eight-ninths, equal to thing. Observe now how much the eight dirhems and eight-ninths are of the money capital, which is twenty dirhems. You will find them to be four-ninths of the same. Take now four-ninths of the measure and also five-ninths of twenty. The value of four-ninths of the measure is twenty-two dirhems and two-ninths; and the five-ninths of the twenty are eleven dirhems and one-ninth. Thus the heirs obtain thirty-three dirhems and one-third, which is as much as two-thirds of the fifty dirhems.—God is the Most Wise!
The neglected state of the manuscript, in which most diacritical points are wanting, makes me very doubtful whether I have correctly understood the author's meaning in several passages of his preface.

In the introductory lines, I have considered the words as an amplification of what might briefly have been expressed by "through the performance of which." I conceive the author to mean, that God has prescribed to man certain duties, and that by performing these (etc.) we express our thankfulness (تكب اسم الشكر) &c.

Since my translation was made, I have had the advantage of consulting Mr. Shakespeare about this passage. He prefers to read , and proposes to translate as follows: "Praise to God for his favours in that which is proper for him from among his laudable deeds, which in the performance of what he has rendered indis-
pensible from (or by reason of) them on (the part of) whoever of his creatures worships him, gives the name of thanksgiving, and secures the increase, and preserves from deterioration."

The construction here assumed is evidently easier than that adopted by myself, in as far as the relative pronoun which representingrovicamade, is made the subject of the three subsequent verbs, &c., whilst my translation presumes a transition from the third person (as in من ما هو أهله, and in نتقع *) to the first (as in نتقع &c.).

A marginal note in the manuscript explains the words لعل تقتديرة ونوع صاحبه من الفيّر by noting from the other side of the page. The meaning may be: we preserve from change him who enjoys it," (viz. the divine bounty, taking صاحب for صاحب صاحب for نعم لله. The change here spoken of is the forfeiture of the divine mercy by bad actions; for "God does not change the mercy which he bestows on men, as long as they do not change that which is within themselves." أبن الله لم يكت مغيرا نعمة انعمها علي توم حتى يغيروا ما بانفسهم (Coran, Sur. viii. v. 55, ed. Hinck.).

Page 1, line 7.

[See Coran, Sur. v. v. 22, ed. Hinck.]

Page 1, line 14, 15.

I am particularly doubtful whether I have correctly read and translated the words of the text from وذكرا to احتسابا. Instead of احتسابا للاجر I should have preferred احسانا.
"benefitting others," if the verb أَحْسَن could be construed with the preposition ل.

Page 2, line 1.

To the words رجل سابق a marginal note is given in the manuscript, which is too much mutilated to be here transcribed, but which mentions the names of several authors who first wrote on certain branches of science, and concludes with asserting, that the author of the present treatise was the first that ever composed a book on Algebra.

Page 2, line 4.

An interlinear note in the manuscript explains نَمَّا سَعَتْهُ جَمْعٌ مَفْتَرْقَةَ by جَمْع مَفْتَرْقَةَ.

Page 2, line 10.

Mohammed gives no definition of the science which he intends to treat of, nor does he explain the words جِبْر jebr, and مُتَقاَبَالَة mokābalah, by which he designates certain operations peculiar to the solution of equations, and which, combined, he repeatedly employs as an expression for this entire branch of mathematics. As the former of these words has, under various shapes, been introduced into the several languages of Europe, and is now universally used as the designation of an important division of mathematical science, I shall here subjoin a few remarks on its original sense, and on its use in Arabic mathematical works.

The verb جِبْر jabar of which the substantive جِبْر jebr is derived, properly signifies to restore something broken,
especially to cure a fractured bone. It is thus used in the following passage from Motanabbi (p. 143, 144, ed. Calcutt.)

"O thou on whom I rely in whatever I hope, with whom I seek refuge from all that I dread; whose bounteous hand seems to me like the sea, as thy gifts are like its pearls: pity the youthfulness of one, whose prime has been wasted by the hand of adversity, and whose bloom has been stifled in the prison. Men will not heal a bone which thou hast broken, nor will they break one which thou hast healed."

Hence the Spanish and Portuguese expression *algebrista* for a person who heals fractures, or sets right a dislocated limb.

In mathematical language, the verb جبر means, to make perfect, or to complete any quantity that is incomplete or liable to a diminution; i. e. when applied to equations, to transpose negative quantities to the opposite side by changing their signs. The negative quantity thus removed is construed with the particle ب: thus, if \( x^2 - 6 = 23 \) shall be changed into \( x^2 = 29 \), the direction is اجبر المال بالسنة وزدها علي الثلثة والعشرين i. e. literally "Restore the square from (the deficiency occasioned to it by) the six, and add these to the twenty-three."
The verb جبر is not likewise used, when in an equation an integer is substituted for a fractional power of the unknown quantity: the proper expression for this is either the second or fourth conjugation of كمل, or the second of تم.

The word مقابلة mokābalah is a noun of action of the verb قبل to be in front of a thing, which in the third conjugation is used in a reciprocal sense of two objects being opposite one another or standing face to face; and in the transitive sense of putting two things face to face, of confronting or comparing two things with one another.

In mathematical language it is employed to express the comparison between positive and negative terms in a compound quantity, and the reduction subsequent to such comparison. Thus \(100 + 10x - 10x + x^2\) is reduced to \(100 + x^2\) بعد أن نقابلنا به "after we have made a comparison."

When applied to equations, it signifies, to take away such quantities as are the same and equal on both sides. Thus the direction for reducing \(x^2 + x = x^2 + 4\) to \(x = 4\) will be expressed by قابل.

In either application the verb requires the preposition ب before a pronoun implying the entire equation or compound quantity, within which the comparison and subsequent reduction is to take place.

The verb قابل is not likewise used, when the reduction of an equation is to be performed by means of a division: the proper term for this operation being خذ. 
The mathematical application of the substantives جبر and مقابلة will appear from the following extracts.

1. A marginal note on one of the first leaves of the Oxford manuscript lays down the following distinction:

"Jebr is the restoration of anything defective by means of what is complete of another kind. Mokābalah, a noun of action of the third conjugation, is the facing a thing: whence it is applied to one praying, who turns his face towards the kiblah. In this branch of calculation, the method commonly employed is the restoring of something defective in its deficiency, and the adding of an amount equal to this restoration to the other side, so as to make the completion (on the one side) and this addition (on the other side) to face (or to balance) one another. As this method is frequently resorted to, it has been named jebr and mokābalah (or Restoring and Balancing), since here every thing is made complete if it is deficient, and the opposite sides are made to balance one another. . . . . . Mathematicians also take
the word *mokābalah* in the sense of the removal of equal quantities (from both sides of an equation)."

According to the first part of this gloss, in reducing $x-5a=10a$ to $x=15a$, the substitution of $x$ in place of $x-5a$ would afford an instance of *jebr* or restoration, and the corresponding addition of $5a$ to $10a$, would be an example of *mokābalah* or balancing. From the following extracts it will be seen, that *mokābalah* is more generally taken in the sense stated last by the gloss.

2. **Haji Khalfa**, in his bibliographical work (MS. of the British Museum, fol. 167, *recto*.) gives the following explanation: "*Jebr* is the adding to one side what is negative on the other side of an equation, owing to a subtraction, so as to equalize them. *Mokābalah* is the removal of what is positive from either sum, so as to make them equal."

A little farther on **Haji Khalfa** gives further illustration of this by an example: "

---

*This manuscript is apparently only an abridgement of Haji Khalfa's work.*
For instance if we say: 'Ten less one thing equal to four things;' then *jebr* is the removal of the subtraction, which is performed by adding to the minuend an amount equal to the subtrahend: hereby the ten are made complete, that which was defective in them being restored. An amount equal to the subtrahend is then added to the other side of the equation: as in the above instance, after the ten have been made complete, one thing must be added to the four things, which thus become five things. *Mokābalah* consists in withdrawing the same amount from quantities of the same kind on both sides of the equation; or as others say, it is the balancing of certain things against others, so as to equalize them. Thus, in the above example, the ten are balanced against the five with a view to equalize them. This science has therefore been called by the name of these two rules, namely, the rule of *jebr* or restoration, and of *mokābalah* or reduction, on account of the frequent use that is made of them.”

3. The following is an extract from a treatise by *Abu Abdallah al-Hosain ben Ahmed,* entitled, [*المقدمة*]

---

* I have not been able to find any information about this writer. The copy of the work to which I refer is comprized in the same volume with *Mohammed ben Musa*’s work in the Bodleian library. It bears no date.
A complete introduction to the elements of algebra.

On the original meaning of the words *jebr* and *mokabalah*. This species of calculation is called *jebr* (or completion) because the question is first brought to an equation ... And as, after the equation has been formed, the practice leads in most instances to equalize something defective with what is not defective, that defective quantity must be completed where it is defective; and an addition of the same amount must be made to what is equalized to it. As this operation is frequently employed (in this kind of calculation), it has been called *jebr*: such is the original meaning of this word, and such the reason why it has been applied to this kind of calculation. *Mokabalah* is the removal of equal magnitudes on both sides (of the equation).

4. In the *Kholāṣet al-Hīsāb*, a compendium of arithmetic and geometry by Baha-eddin Mohammed ben al Hosain (died A.H. 1031, i.e. 1575 A.D.) the Arabic
text of which, together with a Persian commentary by Roshan Ali, was printed at Calcutta* (1812. 8vo.) the following explanation is given (pp. 334. 335.)

The side (of the equation) on which something is to be subtracted, is made complete, and as much is added to the other side: this is jebr; again those cognate quantities which are equal on both sides are removed, and this is mokābahal.

The examples which soon follow, and the solution of which Baha-EdDin shows at full length, afford ample illustration of these definitions. In page 338, $1500 - \frac{1}{4}x = x$ is reduced to $1500 = 1\frac{1}{4}x$; this he says is effected by jebr. In page 341, $7x = \frac{1}{2}x^2 + \frac{1}{2}x$ is reduced to $13x = x^2$, and this he states to be the result of both jebr and mokābahal.

The Persians have borrowed the words jebr and mokābahal, together with the greater part of their mathematical terminology, from the Arabs. The following extract from a short treatise on Algebra in Persian verse, by Mohammed Nadjm-EdDin Khan, appended to the Calcutta edition of the Kholāset al Hisāb, will serve as an illustration of this remark.

* A full account of this work by Mr. Strachey will be found in the twelfth volume of the Asiatic Researches, and in Hutton's Tracts on mathematical and philosophical subjects, vol. ii. pp. 179-193. See also Hutton's Mathematical Dictionary, art. Algebra.
Complete the side in which the expression illā (less, minus) occurs, and add as much to the other side, O learned man: this is in correct language called jebr. In making the equation mark this: it may happen that some terms are cognate and equal on each side, without distinction; these you must on both sides remove, and this you call mokābalah."

With the knowledge of Algebra, its Arabic name was introduced into Europe. Leonardo Bonacci of Pisa, when beginning to treat of it in the third part of his treatise of arithmetic, says: Incipit pars tertia de solutione quarundam questionum secundum modum Algebrae et Almucabala, scilicet oppositionis et restaurationis. That the sense of the Arabic terms is here given in the inverted order, has been remarked by Cossali. The definitions of jebr and mokābalah given by another early Italian
writer, Lucas Pacioli, or Lucas de Burgo, are thus reported by Cossali: Il commune oggetto dell' operar loro è recare la equazione alla sua maggior unità. Gli uffizj loro per questo commune intento sono contrarj: quello dell' Algebra è di restorare li extremi dei diminuti; e quello di Almucabala di levare da li extremi i superflui. Intende Fra Luca per extremi i membri dell' equazione.

Since the commencement of the sixteenth century, the word mokābalah does no longer appear in the title of Algebraic works. Hieronymus Cardan's Latin treatise, first published in 1545, is inscribed: Artis magnæ sive de regulis algebraicis liber unus. A work by John Scheubelius, printed at Paris in 1552, is entitled: Algebra compendiosa facilisque descriptio, qua depromuntur magna Arithmetices miracula. (See Hutton's Tracts, &c. ii. pp. 241-243.) Pelletier's Algebra appeared at Paris in 1558, under the title: De occulta parte numerorum quam Algebrae vacant, libri duo. (Hutton, l. c. p. 245. Montucla, hist. des math. i. p. 613.) A Portuguese treatise, by Pedro Nuñez or Nonius, printed at Amberez in 1567, is entitled: Libro de Algebra y Arithmetica y Geometria. (Montucla, l. c. p. 615.)

In Feizi's Persian translation of the Lilavati (written in 1587, printed for the first time at Calcutta in 1827, 8vo.) I do not recollect ever to have met with the word جبر; but مقابلته is several times used in the same sense as in the above Persian extract.
Page 3, line 3, seqq.

In the formation of the numerals, the thousand is not, like the ten and the hundred, multiplied by the units only, but likewise by any number of a higher order, such as tens and hundreds: there being no special words in Arabic (as is the case in Sanscrit) for ten-thousand, hundred-thousand, &c.

From this passage, and another on page 10, it would appear that our author uses the wordعقد, plur. عقود, knot or tie, as a general expression for all numerals of a higher order than that of the units. Baron S. de Sacy, in his Arabic Grammar, (vol. i. § 741) when explaining the terms of Arabic grammar relative to numerals, translatesعقد bynœuds, and remarks: Ce sont les noms desdixaines, depuis vingt jusqu'à quatre-vingt-dix.

Page 3, line 9-11.

The forms of algebraic expression employed by Leonardo are thus reported by Cossali (Origine, &c. dell'Algebra, i. p. 1.): Tre considerazioni distinguendo Leonardo nel numero: una assoluta, o semplice, ed è quella del numero in se stesso; le altre due relative, e sono quelle di radice e di quadrato. Nominando il quadrato suggeste qui videlicet census dicitur, ed il nome di censo è quello di cui in seguito si serve. That Leonardo seems to have chosen the expression census on account of its acceptance, which is correspondent to that of the
Arabic َّال, has already been remarked by Mr. Colebrooke (Algebra, &c., Dissertation, p. liv.)

Paciolo, who wrote in Italian, used the words numero, cosa, and censo; and this notation was retained by Tartaglia. From the term cosa for the unknown number, exactly corresponding in its acceptation to the Arabic شيء thing, are derived the expressions Ars cossica and the German die Coss, both ancient names of the science of Algebra. Cardan's Latin terminology is numerus, quadratum, and res, for the latter also positio or quantitas ignota.

Page 3, line 17.

I have added from conjecture the words which are not in the manuscript. There occur several instances of such omissions in the work.

The order in which our author treats of the simple equations is, 1st. \( x^2 = px \); 2d. \( x^2 = n \); 3d. \( px = n \). Leonardo had them in the same order. (See Cossali, l. c. p. 2.) In the Kholāṣet al Hisāb the arrangement is, 1st. \( n = px \); 2d. \( px = x^2 \); 3d. \( n = x^2 \).

Page 5, line 9.

In the Lilavati, the rule for the solution of the case \( cx^2 + bx = a \) is expressed in the following stanza.
i.e. rendered literally into Latin:

Per multiplicatam radicem diminutæ [vel] auctæ quantitatis

Manifestæ, additæ ad dimidiati multiplicatoris quadratum

Radix, dimidiato multiplicatore addito [vel] subtracto,

In quadratum ducta—est interrogantis desiderata quantitas.

The same is afterwards explained in prose: यो राशि: स्वमूलेन केनचित् गुणितेन उनो युतो वा दृष्टस्तस्य भूलस्य गुणार्थक्या युतस्य दृष्टस्य यतू पदं तदनाधिरेन युतं यदि मूलोनो दृष्टो राशिभेवति यदि गुणार्थभूलयतो दृष्टस्तर्हि विहीनं कार्यी तत्स्य वर्गों राशि: स्यात् ॥ i.e. “A quantity, increased or diminished by its square-root multiplied by some number, is given. Then add the square of half the multiplier of the root to the given number: and extract the square-root of the sum. Add half the multiplier, if the difference were given; or subtract it, if the sum were so. The square of the result will be the quantity sought.” (Mr. Colebrooke’s translation.)

Feizi’s Persian translation of this passage runs thus:

مرگاه شخصی عدیدی را منصور کر وجدز او را یا کسی
With the above Sanskrit stanza from the Lilavati some readers will perhaps be interested to compare the following Latin verses, which Montucla (i. p. 590) quotes from Lucas Paciolus:

Si res et census numero coaequantur, a rebus
Dimidio sumpto, census producere debes,
Adderque numero, cujus a radice totiens
Tolle semis rerum, census latusque redibit.

Page 6, line 16.

Such instances of the common instead of the apocopate future, after the imperative, are too frequent in this work, than that they could be ascribed to a mere mistake of the copyist: I have accordingly given them as I found them in the manuscript.
Page 7, line 1.

The same structure occurs page 21, line 15.

Page 8, line 11.

Hadji Khalfa, in his article on Algebra, quotes the following observation from Ibn Khaldun. قال ابن خلدون وقد بلغنا أن بعض أئمة التعليم من أهل المشرق انتهج المعادلات التي أكثر من هذه السنة وبلغها الي فرق العشرين واستخرج لها كلها أعماله وثيقة براهين هندسية.

"Ibn Khaldun remarks: A report has reached us, that some great scholars of the east have increased the number of cases beyond six, and have brought them to upwards of twenty, producing their accurate solutions together with geometrical demonstrations."

Page 8, line 17.

See Leonardo's geometrical illustration of the three cases involving an affected square, as reported by Cossali (I. p. 2.), and hence by Hutton (Tracts, &c., ii. p. 198.)

Cardan, in the introduction of his Ars Magna, distinctly refers to the demonstrations of the three cases given by our author, and distinguishes them from others which are his own. At etiam demonstrationes, præter tres Mahometis et duas Lodovici (Lewis Ferrari, Cardan's pupil), omnes nostræ sunt.—In another passage (page 20) he blames our author for having given the demonstration of only one solution of the case $cx^2 + a = bx$. Nec admireris,
says he, *hanc secundam demonstrationem aliter quam a Mahumete explicantem, nam ille immutata figura magis ex re ostendit, sed tamem obscurius, nec nisi unam partem eamque pluribus.*

*Page 17, line 11-13.*

The words from وسَدُس السَّدَسْ to والا سَدَسَا في are written twice over in the manuscript.

*Page 19, line 12.*

جذر مال معلوم أو اسم ] “The root of a rational or irrational number.” In the *Kholāṣet al Hisāb*, p. 128. 137. 369, the expression مطلق (lit. audible) is used instead of اسم معلوم, which stands in a more distinct opposition to اسم (lit. inaudible, surd). Baha-eddin applies the same expressions also to fractions, calling مطلق those for which there are peculiar expressions in Arabic, e. g. خَلْف one-third, and اسم those which must be expressed periphrastically by means of the word جزء a part, e. g. خَلْف اجزاء من خمسة وعشرين three twenty-fifths. See *Kholāṣet al Hisāb*, p. 150.

*Page 19, line 15.*

The manuscript has مثلي ذلك المال. The context requires the insertion of the word جذر after مثلي which I have added from conjecture.

*Page 20, line 15. 17.*

ما يصيب الواحد ] “What is proportionate to the unit,”
i.e. the quotient. This expression will be explained by Baha-eddin's definition of division (Kholāset al Hisāb, p. 105). Division is the finding a number which bears the same proportion to the unit, as the dividend bears to the divisor.

Page 21, line 17.

The MS. has [جذري].

Page 24, line 6.

An attempt at constructing a figure to illustrate the case of \[100 + x^2 - 20x + [50 + 10x - 2x^2]\] has been made on the margin of the manuscript.

Page 30, line 10.

A marginal note in the manuscript defines this in the following manner. He means to say: divide the ten in any manner you like, taking four of wheat and six of barley, or four of barley and six of wheat, or three of wheat and seven of barley, or vice versa, or in any other way: for the solution will hold good in all these cases. (Note from Al Mozaihafi's Commentary).

Page 42, line 8.

The manuscript has a marginal note to this passage, 2 c
from which it appears that the inconvenience attending the solution of this problem has already been felt by Arabic readers of the work.

Page 45, line 16.

This instance from Mohammed's work is quoted by Cardan (Ars Magna, p. 22, edit. Basil.) As the passage is of some interest in ascertaining the identity of the present work with that considered as Mohammed's production by the early propagators of Algebra in Europe, I will here insert part of it. Nunc autem, says Cardan, subjungemus aliquas questiones, duas ex Mauumete, reliquas nostras. Then follows Quæstio I. Est numerus a cujus quadrato si abjeceris $\frac{1}{2}$ et $\frac{1}{4}$ ipsius quadrati, atque insuper 4, residuum autem in se duxeris, fiet productum æquale quadrato illius numeri et etiam 12. Pones itaque quadratum numeri incogniti quem quæris esse 1 rem, abjice $\frac{1}{3}$ et $\frac{1}{4}$ ejus, es insuper 4, fiet $\frac{5}{12}$ rei m: 4, duc in se, fit $\frac{25}{144}$ quadrati p: 16 m: $3\frac{1}{3}$ rebus, et hoc est æquali uni rei et 12; abjice similia, fiet 1 res æqualis $\frac{25}{144}$ quadrati p: 4 m: $3\frac{1}{3}$ rebus, &c.

The problem of the Quæstio II. is in the following terms, Fuerunt duo duces quorum unusquisque divisit militibus suis aurcos 48. Porro unus ex his habuit milites duos plus altero, el illi qui milites habuit duos minus contigit ut aurcos quatuor plus singulis militibus daret; quæritur quot unicuique milites fuerint. In the present copy of Mohammed's algebra, no such instance occurs. Yet Car-
DAN distinctly intimates that he derived it from our author, by introducing the problem which immediately follows it, with the words: *Nunc autem proponamus quæstiones nostras.*

Page 46, line 18.

The manuscript has the following marginal note to this passage:

This instance may also be solved by means of a cube. The computation then is, that you take the square, and remove one-third from it; there remain two-thirds of a square. Multiply this by three roots; you find two cubes equal to one square. Extracting twice the square-root of this, it will be two roots equal to a dirhem. Accordingly one root is one-half, and the square one-fourth.* If you remove one-third of this, there remains one-sixth, and if you multiply this by three roots, that is by one dirhem and a half, it amounts to one-fourth of a dirhem, which is the square as he had stated.”

* \[ (x^2 - \frac{1}{3}x^2) \times 3x = x^2 \]

\[
2x^3 = x^2
\]

\[
2x = 1
\]

\[
x = \frac{1}{2}.
\]
Page 50, line 2.

I am uncertain whether my translation of the definition which Mohammed gives of mensuration be correct. Though the diacritical points are partly wanting in the manuscript, there can, I believe, be no doubt as to the reading of the passage.

Page 51, line 12.

I have simply translated the words اهل الهندسة by "geometricians," though from the manner in which Mohammed here uses that expression it would appear that he took it in a more specific sense.

Firuzabadi (Kamus, p. 814, ed. Calcutt.) says that the word handasah (الأندسة) is originally Persian, and that it signifies "the determining by measurement where canals for water shall be dug."

The Persians themselves assign yet another meaning to the word Hindisah, as they pronounce it: they use it in the sense of decimal notation of numerals.*

It is a fact well known, and admitted by the Arabs

هندسة بكرسول وثالث وفتح سين بيب نقطه بمعنى
اندزة وشكل باشد وإراقية را نيزغوبند كه در زير حروف
كلمات نويسند همچون السبند هو ز حطي
1 8 7 6 5 4 3 2 1

"Hindisah is used in the sense of measurement and size; the same word is also applied to the signs which are written instead of the words (for numbers) as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10." Burhani Kati.
themselves, that the decimal notation is a discovery for which they are indebted to the Hindus.* At what time the communication took place, has, I believe, never yet been ascertained. But it seems natural to suppose that it was at the same period, when, after the accession of the Abbaside dynasty to the caliphat, a most lively interest for mathematical and astronomical science first arose among the Arabs. Not only the most important foreign works on these sciences were then translated into Arabic, but learned foreigners even lived at the court of Bagdad, and held conspicuous situations in those scientific establishments which the noble ardour of the caliphs had called forth. History has transmitted to us the names of several distinguished scholars, neither Arabs by birth nor Mohammedans by their profession, who were thus attached to the court of Almansur and Almamun; and we know from

* It is almost unnecessary to adduce further evidence in support of this remark. Baha-Ed- din, after a few preliminary remarks on numbers, says "Learned Hindus have invented the well known nine figures for them." (Kholâset al-Hisâb, p. 16.) In a treatise on arithmetic, entitled "Mathematics in Science of the Hindus", which forms part of Sir W. Ouseley's most valuable collection of Oriental manuscripts, the nine figures are simply called the symbols of the Hindu. See, on the subject generally, Professor von Böhlen's work, Das alte Indien, (Königsberg, 1830. 1831. 8.) vol. ii. p. 224, and Alexander von Humboldt's most interesting dissertation: Ueber die bei verschiedenen Völkern üblichen Systeme von Zahlzeichen, &c. (Berlin, 1829. 4.) page 24.
good authority, that Hindu mathematicians and astronomers were among their number.

If we presume that the Arabic word *handasah* might, as the Persian *hindisah*, be taken in the sense of decimal notation, the passage now before us will appear in an entirely new light. The *أهل الهندسة*, to whom our author ascribes two particular formulas for finding the circumference of a circle from its diameter, will then appear to be the Hindu Mathematicians who had brought the decimal notation with them;—and the *أهل التحوم منهم*, to whom the second and most accurate of these methods is attributed, will be the Astronomers among these Hindu Mathematicians.

This conjecture is singularly supported by the curious fact, that the two methods here ascribed by Mohammed to the *أهل الهندسة* actually do occur in ancient Sanskrit mathematical works. The first formula, \( p = \sqrt{10d^2} \), occurs in the *Vijaganita* (Colebrooke’s translation, p. 308, 309.); the second, \( p = \frac{d \times 63832}{20000} \), is reducible to \( \frac{d \times 3927}{1250} \), the proportion given in the following stanza of Bhaskara’s *Lilavati*:

\[
\begin{align*}
\text{वासे भन्द्राग्रिहने चिन्हने} & \\
\text{खवाणसूयें: परिधिस्तु सूक्ष्म:} & 1 \\
\text{दाविंशतिनिघे चिन्हने च शैले:} & \\
\text{स्थूले श्च वा स्याढवहारयोग्य:} & 11
\end{align*}
\]

When the diameter of a circle is multiplied by three
thousand nine hundred and twenty-seven, and divided by twelve hundred and fifty, the quotient is the near circumference: or multiplied by twenty-two and divided by seven, it is the gross circumference adapted to practice."* (Colebrooke's translation, page 87. See Feizi's Persian translation, p. 126, 127.)

The coincidence of \(\frac{d \times 62832}{20000}\) with \(\frac{d \times 3927}{1250}\) is so striking, and the formula is at the same time so accurate, that it seems extremely improbable that the Arabs should by mere accident have discovered the same proportion as the Hindus: particularly if we bear in mind, that the Arabs themselves do not seem to have troubled themselves much about finding an exact method.†

* The Sanskrit original of this passage affords an instance of the figurative method of the Hindus of expressing numbers by the names of objects of which a certain number is known: the expressions for the units and the lower ranks of numbers always preceding those for the higher ones. भ (lunar mansion) stands for 27; नन्द (treasure of Kuvera) for 9; and अग्रि (sacred fire) for 3: therefore भनन्दा-ग्रि = 3927. Again, ख (cypher) is 0; वाण (arrow of Kamadeva) stands for 5; सूरि (the sun in the several months of the year), for 12: therefore खवाणसूरि = 1250. For further examples, see As. Res. vol. xii. p. 281, ed. Calc., and the title-pages or conclusions of several of the Sanskrit works printed at Calcutta;—e. g. the Sutras of Panini and the Siddhantakaumudi.

† This would appear from the very manner in which our author introduces the several methods; but still more from the following marginal note of the manuscript to the present passage: ورد تقربب
Page 57, line 5-8.

The words between brackets are not in the manuscript: I have supplied the apparent hiatus from conjecture.

Page 61, line 4.

A triangle of the same proportion is used to illustrate this case in the Lilavati (Feizi's Persian transl. p. 121. Colebrooke's transl. of the Lilavati, p. 71. and of the Vijaganita, p. 203.)

Page 65, line 12-14.

The words between brackets are in the manuscript written on the margin. I think that the context warrants me sufficiently for having received them into the text.

Page 66, line 5.

The words between brackets are not in the text, I give them merely from my own conjecture.

لا تحقيق ولا يقف احد علي حقيقة ذلك ولا يعلم دوردا الا لله لا يعتقد بمستقيم فيذهب علي حقيقة وإنما قبل ذلك تقرب كما قيل في جذر الاسم أنه تقرب لا تحقيق لأن جذر لا يعلم الا الله وأحسن ما في هذه الايام أن تصرف النظر في ثالث وسبع لأنه أخف وأسرع والله اعلم.

"This is an approximation, not the exact truth itself: nobody can ascertain the exact truth of this, and find the real circumference, except the Omniscient: for the line is not straight so that its exact length might be found. This is called an approximation, in the same manner as it is said of the square-roots of irrational numbers that they are an approximation, and not the exact truth: for God alone knows what the exact root is. The best method here given is, that you multiply the diameter by three and one-seventh: for it is the easiest and quickest. God knows best!"
The author says, that the capital must be divided into 219320 parts: this I considered faulty, and altered it in my translation into 964080, to make it agree with the computation furnished in the note. But having recently had an opportunity of re-examining the Oxford manuscript, I perceive from the copious marginal notes appended to this passage, that even among the Arabian readers considerable variety of opinion must have existed as to the common denominator, by means of which the several shares of the capital in this case may be expressed.

One says: انظر لمال يكون لسدسه ربع والربيع ثلث وما بقي يتقسم علي مائة وخمسة وتسعين ولا يوجد ذلك في اتل من اربعة وعشرين فاضرب اربعة وعشرين في مائة وخمسة و تسعين بسج من ذلك اربعة آلاف وستمائة وثمانون ومنه يصح “Find a number, one-sixth of which may be divided into fourths, and one-fourth of which may be divided into thirds; and what thus comes forth let be divisible by hundred and ninety-five. This you cannot accomplish with any number less than twenty-four. Multiply twenty-four by one hundred and ninety-five: you obtain four thousand six hundred and eighty, and this will answer the purpose.”

Another: وفي وجه اخر انك تجعل مائة وستة وخمسين

* The numbers in this and in part of the following scholium are in the MS. expressed by figures, which are never used in the text of the work.
According to another method, you may take one hundred and fifty-six for the one-sixth of the capital. Multiply this by six; you find nine hundred and thirty-six. Taking from this the share of the son, which is one-third and one-fourth, you find it five hundred and forty-six. This is not divisible by five: therefore multiply the whole number of parts by five: it will then be four thousand six hundred and eighty. Of this the mother receives four hundred and twenty-five, the husband seven hundred and eighty, the son two hundred and eighty-eight (twelve hundred and eighty-eight?), the legatee, who is to receive the two-fifths, fourteen hundred and ninety-two, and the legatee to whom the one-fourth is bequeathed, six hundred and ninety-five."

Another: "وفي [وجه] آخر يصح من نسمة الاف وثلثماية وستين ووجه العمل في ذلك ان [تقسم] الفريضة في اثني عشر للمهم والزوج ثلثة وللابن سبع ضريرها في 20 لذكر الخمسي والربع يكون مائتين وأربعين فتاخذها سدسًا اربعين للمهم والثلث جائز عليها وليس للربعين ثلث ضرير اصل المسألة في ثلثة لذلك يكون سبعا وعشرين وضيعة فتاخذ سدسًا للمهم مائة وعشرين فتخرج من ذلك الثلث لإصحاب الوصايا وهو اربعون مقسم علي ثلثة عشر لا يصح فاضرب المسألة في
According to another method, the number of parts is nine thousand three hundred and sixty. The computation then is, that you divide the property left into twelve shares; of these the mother receives two, the husband three, and the son seven. This (number of parts) you multiply by twenty, since two-fifths and one-fourth are required by the statement. Thus you find two hundred and forty. Take the sixth of this, namely forty, for the mother. One-third out of this she must give up. Now, forty is not divisible by three. You accordingly multiply the whole number of parts by three, which makes them seven hundred and twenty. The one-sixth of this for the mother is one hundred and twenty. One-third of this, namely forty, goes to the legatees, and should be divided by thirteen; but as this is impossible, you multiply the whole number of parts by thirteen, which makes them nine thousand three hundred and sixty, as we said above. Of this the mother receives eight hundred and fifty, the son two thousand five hundred and seventy-six, the husband one thousand five hundred and sixty, the legatee to whom the two-fifths are bequeathed, two thousand nine hundred and eighty-four, and the legatee who is to receive one-fourth, one thousand three hundred and ninety.
there remains nine of it, and this is the deduction from the completement. Subtracting it from the completement, which is thirteen, there remains four, and this is the legacy, as the author has said."

Page 98, line 8.

The word مثلاً which I have omitted in my translation of this and of two following passages, is in the manuscript explained by the following scholium:

"Adequate, i. e. corresponding to her beauty, her age, her family, her fortune, her country, the state of the times, .... and her virginity." (Part of the gloss is to me illegible.) The dowry varies according to any difference in all the circumstances referred to by the scholium. See Hamilton's Hedaya, vol. i. page 148.

Page 113, line 7.

The manuscript has the following marginal note (?). "The Okr of a slave girl corresponds to the adequate dowry of a free-born woman; it is a sum of money on payment of which one of distinguished qualities corresponding to her would be married." See Hamilton's Hedaya, vol. ii. page 71.

I am very doubtful whether I have well understood the words in which our author quotes Abu Hanifah's opinion. Abu Hanifah al No'man ben Thabet is well known
as an old Mohammedan lawyer of high authority. He was born at Kufa, A.H. 80 (A.D. 690), and died A.H. 150 (A.D. 767). Ebn Khallikan has given a full account of his life, and relates some interesting anecdotes of him which bear testimony to the integrity and independence of his character.

Page 113, line 16.

The marginal notes on this chapter of the manuscript give an account of what the computation of the cases here related would be according to the precepts of different Arabian lawyers, e.g. Shafei, Abu Yussuf, &c. The following extract of a note on the second case will be sufficient as a specimen:

الجواب الذي ذكره الخوارزمي
في هذه المسألة ائتم هو علي مذهب أبي يوسف وزفر (*)
واحد الوجه لإصحاب الشافعي فاما أبو حنيفة فانه يجعل ما
لزم الوهاب بسبب وطنه وصية ائتما فتكون الوصية علي قوله
شيعا وثلثا وهو أحد الوجه علي مذهب الشافعي وعند محمد
بن الجهمي (*) يجعل وط الوهاب لما وهب منه ولا يلزمه
شيء بسبب ذلك وهو أحد الوجه علي هذهب الشافعي
فعلي هذا الوجه تصح البيئة في ثلثي ولا تبطل في ثلثي ولا دوران
التركة علي حالبا والي قول ابن حنفية تعمل لما فعلت علي
مذهب أبي يوسف وزفر (**) فإذا صار بائدي الورثة ثلثماية الا
شيعا وثلث شوي، يعدل شيئين وثلثي شيء للذوي لومة
العفر وصية ائتما فإذا جربت وثابات عمل الشيء خمسة
وعمرين ديهما وهو ربع للأجارة فتصح البيئة في ربعها وتبطل

(*) These names are very indistinctly written in the manuscript.
The solution of this question given by the Khowarezmian is according to the school of Abu Yussuf Wazfar, and one of the methods of Shafei's followers. Abu Hanifah calls the sum which the donor has to pay on account of having cohabited with the slave-girl likewise a legacy; thus, according to him, the legacy is one and one-third of thing: this is another method of Shafei's school. According to Mohammed ben al Jaish, the donor has nothing to pay on account of having cohabited with the slave girl:* and this is again a method adopted by the school of Shafei. After this method, one-third of the donation is really paid, whilst two-thirds become extinct: and there is no return, as the heritage has remained unchanged. According to Abu Hanifah, you proceed in the same manner as after the precepts of Abu Yussuf Wazfar. Thus the heirs obtain three hundred less one and one-third of thing, which is equal to two things and two-thirds: for what he (the donor) has to pay on behalf of the dowry, is likewise a legacy. Completing and reducing this, one thing is equal to seventy-five dirhems: this is one-fourth for the slave-girl; one-fourth of the donation is actually paid, and three-fourths become extinct.”

* I doubt whether this is the meaning of the original, the words from ل 소개حم till بانه being very indistinctly written in the MS.
<table>
<thead>
<tr>
<th>سطر</th>
<th>خمسة نفرد</th>
<th>خمسة نفر</th>
<th>خمسة نفر</th>
<th>خمسة نفرد</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>2</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>3</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>4</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>5</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>6</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>7</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>8</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>9</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>10</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>11</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>12</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>13</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>14</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>16</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>17</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>18</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>19</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>20</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>21</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>22</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>23</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>24</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>25</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>26</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>27</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>28</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>29</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>30</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
<td>1111</td>
</tr>
</tbody>
</table>

الصحيح
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
-
١٣٣

درهم وشي ونصف شي نصف نصفها هو الوصية وهو عشرة درهم وثلثة أرباع شي وذالك ثلث المال وهو ستة عشر درهم وثلثا دهم فالتح عشرة بعشرة فيقي ستة درهم وثلثان يعدل ثلاثة أرباع شي نكمل الشي وهو ان تزيد عليه ثلاثة وزن علي السنة والثلثين ثلثها وهو درهمان وتسعا درهم فيكون ثمانية درهم وثمانية انساع درهم يعدل شيئا فانظركم الثمانية الدراهم والثمانية الاتساع من رأس المال وهو عشرون درهما فتجد ذلك اربعة انساعها فرد من الكر اربعة انساعة وترد خمسة انساع العشرين فيكون قيمة اربعة انساع الكر ثاني وعشرين درهما وتسع درهم وخمسة انساع العشرين احد عشر درهما وتسع درهم فيصير في ايدي الورثة ثلاثة وثلثين درهما وثلث درهم وهو ثلثا الخمسين الدراهم * والله اعلم *

* تم الكتاب بحمد الله ومنه وتوفيقه وتشديده
باب السلم في المرض

إذا أسلم رجل في مرضه ثلاثين درهما في كر من طعام يساوي عشرة دراهم ثم مات في مرضه فانه يرد الكر ويعد
علي ورثة الميت عشرة دراهم قياسه ان يرد الكر وقيمه
عشرة دراهم فهو قد حاباه بعشرين درهما فالوصية من
المهاباة شي ويصير في ايدي الوثرة عشرون غير شي.
وكر وكل ذلك ثلاثون درهما غير شي يعدل شهين وهو
مثلا الوصية فاجبر الثلاثين بالشي وزده علي الشهيد قصير
الثلاثون يعدل ثلاثة اشياء الشي من ذلك ثلث وهو عشرة
德拉هم وهوما جازمس المهاباة

فإن أسلم الي رجل عشرين درهما وهو مريض في كر
يساوي خمسين درهما ثم اقلاة في مرضه ثم مات فانه يرد
اربعا اتباع الكر وأحد عشر درهما وتسع درهم وقيسه
انك قد علمت ان قيمة الكر مثل الذي أسلم إليه
مرتين ونصفا فهو لا يرد من رأس المال شيئ إلا رد من
الكر مثلي ومثل نصفه فتجعل الذي يرد من الكر بالشي
فشيئين نصفا فنفذه علي ما بقي من العشرين وهو
عشرون غير شي فيصير في ايدي ورثة الميت عشرون
فيكون بعض الشيء وثلثين درهماً يعدل نصف شيء.
فيكون نصف شيء غير ثلثين يعدل بعض الشيء الذي هو وصية المهوب له للواعب فأعرف ذلك ثم ارجع الي ما بقي في يد الواعب وهو ثلثمياً غير شيء وصار اليه بعض الشيء وهو نصف الشيء الا ثلثين درهماً فيبقي في يده مائتان وسبعون غير نصف شيء واخذ العقر وهو مائة درهم غير ثلثين شيء ورد العقر وهو ثلث ما بقي من الشيء بعد رفع بعض الشيء منه وهو سدس شيء وعشرة دراهم فحصل في يده ثلثمياً وستون غير شيء وذلك مثلاً الشيء والعقر الذي رَت نصف ذلك مائة وثمانون غير نصف شيء وهو مثل الشيء والعقر فاجبر ذلك بنصف شيء وزده على الشيء والعقر فيكون مائة ومائتين درهماً يعدل شيئاً ونصف شيء والعقر الذي رَت وهو سدس شيء وعشرة دراهم تستقر عشرة بعشرة فيبقي مائة وسبعون. درهماً يعدل شيئاً وثلثي شيء فأرادنا لتعرف الشيء وهو أن تأخذ ثلثة اخمسة فيكون مائة وأثنين يعدل الشيء الذي هو وصية الواعب للمهوب له وأما وصية المهوب له للواعب فهو نصف ذلك غير ثلثين درهماً وهو واحد وعشرون والله أعلم.
شأن و نثل شي فاجر ذلك بثلاهة اشياء فيكون اربعمية
بالذي الواحد يعدل مهاني واربعين درهما
فإن قال رجل وهب لرجل جارية في مرده قيمتهن
ثلثماية درهم وعشرها من درهم نوطنها الموهوب له ثم
وهنا الموهوب له للواهب في مرده ايضا نوطنه الواهب
فإن زج منها وكم انفق قياسه أن تجعل قيمتها ثلاثماية
درهم وواصية من ذلك شي فيبقي في ايدي ورثة
الواهب ثلاثماية غير شيء وصار في يد الموهوب له شيء
واعطى الموهوب له الواهب بعض الشيء و بقي في يده
شي غير بعض شيء و عن اليه مائة غير نثل شيء
واخذ العقر نثل شئ غير نثل بعض شيء فصار في يده
شي ونثل شيء غير مائة درهم وغير بعض شيء و غير
نثل بعض الشيء و ذلك مثل بعض الشيء فنصفت
مثل بعض الشيء وهو خمسة اساداس شيء غير خمسين
درهما و غير نثل بعض شيء فاجر ذلك بثلاهة بعض
الشي و خمسين درهما فيكون خمسة اساداس شيء تعد
بعض شيء و نثل بعض شيء و خمسين درهما فاردد
ذلك الي بعض شيء لنعرفه وهو ان تأخذ ثلثا الحماس
بتلك ماله: فان قول أبي حنيفة الثالث بينهما نصفان وقياسه ان يجعل الوصية للموهوب له الجارية شيئا فائقا في الثلثاءة غير شيء ثم رقه العقر وهو ثلث شيء فائقا معه في الثلثاءة غير شيء وثلث شيء فوصية في قول أبي حنيفة شيء وثلث شيء وفي قول الآخر شيء ثم تعطي الموسي له بالثلث مثل وصية الأول وهو شيء وثلث شيء فائقا في يده في الثلثاءة غير شيئا وثلثي شيء يعدل مثلي الوصتين رهما شئان وثلثا شيء فنصف ذاك يعدل الوصتين وهو ماية وخمسة غير شيء وثلث شيء فاجبر ذاك بشيء وثلث شيء ورد على الوصتين فصار ماية وخمسين يعدل أربعة أشياء فالشيء من ذاك ربع وهو سبعة وثلاثون ونصفا

فان قال ولائها المودوب له ووقتها الواهب وأوصى بتك ماله * فان التباس في قول أبي حنيفة أن يجعل الوصية شيئا فائقا في الثلثاءة غير شيء وواخذ العقر ماية غير ثلاثة شيء فصار في يده ارباعية درهم غير شيء وثلث شيء ورقة العقر ثلاثة شيء واعطا الموسي له بالثلث مثل وصية الأول شيئا وثلث شيء فائقا ارباعية درهم غير ثلاثة أشياء يعدل مثلي الوصية وذلك
فصار في إدي ورثة الواهب تلثماية غير شيء وثلث شيء، وذلك مثل الوصية التي هي شيء وهو شيئان فاجر ذلك شيء وثلث شيء، وزده علي الشهرين فيكون تلثماية يعدل ثلاثة اشياء وثلث شيء فالشيء من ذلك ثلاثة اشتعار وهو تسعون درهماً وذاك الوصية.

فإن كانت المسألة على حالها وطقها الواهب والموهوب للقياسة أن تجعل الوصية شيئًا والمتقص تلثماية غير شيء ويلزم الواهب للموهوب له العقر بالوطبيء ثلث شيء ويلزم الموهوب له ثلث الانتقاص وهو ماية غير ثلث شيء فصار في إدي ورثة الواهب اربعمية غير شيء وثلثي شيء وذاك مثل الوصية فاجر الربعمية بشيء وثلثي شيء وزدها علي الشهرين فيكون اربعمية يعدل ثلاثة اشياء وثلثي شيء فالشيء من ذلك ثلاثة أجزاء من أحد عشر جزءاً من اربعمية وهو ماية وتسعة وجزء من أحد عشر من درهم وذلك الوصية والانتقاص مائة وتسعون وعشرة أجزاء من أحد عشر جزءاً من درهم. وفي قول أبي حنيفة تجعل الشيء وصية وما أشار إليه بالعقر ابنا وصية.

فإن كانت المسألة على حالها فوطني الواهب وأسمي
درهم يعدل شيئين وتسعة وعشرين جزءاً من أربعين جزءاً من شيء، فقابل به فيكون الشيء يعدل ثلثة وسبعين درهماً وثلثة واربعين جزءاً من مائة وتسعة أجزاء من درهم

* باب العقر في الدور *

رجل وعهب لرجل جارية في مرض موتها ولا مال له غيرها، ثم مات وقيمتها تلمايماً درهم وعقرها مائة درهم فوطنتها الرجل الموحوب له فقياسه أن يجعل الوصية الموهوب له الجارية شيئاً فتنقص منه اللهمة تلمويمه غير شيء، ويرجع إلى ورثة الوابث ثلث الأنتقي للعقر لأن العقر ثلث القيمة وذلك مائة درهم خير ثلث شيء فصار في أيدي ورثة الوابث ارعمية خير شيء وثلث شيء وذلك مثل الوصية التي هي شيء وذلك شيئان فاجر الارعمية بشيء وثلث شيء وزده على الشئين فيكون ارعمية يعدل ثلاثة أشياء وثلث شيء، وشيء من ذلك ثلثة اعشار وهو مائة وعشرون درهماً وهي الوصية.

* قال ويها في مرض وقيمتها تلمايماً وعقرها مائة* 

فوطنها الوابث ثم مات فقياسه أن يجعل الوصية شيئاً والمنقص لتلمويمه غير شيء، فوطنها الوابث فلفزمت العقر وهو ثلث القيمة لأن العقر ثلث القيمة وهو ثلث شيء.
عشرة أجزاء من واحد وثلثين جزءاً من درهم فلالوصية
من المائتين علي قدر ذلك وهي ارבעة وستون درهماً
وستة عشر جزءاً من واحد وثلثين جزءاً من الدهم *
فإن اعتق جارية قيمتها ماية درهم وهوب لرحل
جارية قيمتها خمسماية درهم فوائديها الموهوب له وعقرها
ماية درهم ورعي الواهب لرحل بريع ماله فقول أبي
حنيفة إن صاحب الجارية لا يضرب بكثر من الثلاث
وصاحب الربع يضرب بالربع * وقياس ان قيمة
الجارية خمسماية درهم والوصية من ذلك شيء فيبيقي
خمسماية درهم غير شيء واحد و العشرة ستمائة درهم غير
خمس شيء فصار في أيدي الورثة ستمائة درهم غير شيء
وخمس شيء ثم تعزل وصية صاحب الربع ثلاثة ارباع
شيء لما الثالث إذا كان شيئاً فالربع ثلاثة ارباع شيء
فيبقى ستمائة درهم غير شيء وثمانية وثلثين جزءاً من اربعين
جزء من شيء وذلك مثل الوصية فنصف ذلك يعدل
وصاياهم وهي ثلثماية درهم غير تسعة وثلثين جزءاً من
اربعين جزءاً من شيء فأجزي ذلك بعده الإجزاء فيكون
ثلثماية درهم يعدل مائة درهم وسبعين وتسعة وعشرين
جزءاً من اربعين جزءاً من شيء فاطرح مائة ثانية فيبيقي مائتا
فقال بذلك فتجد الشيء من ذلك خمسة أثمانية
فتخذ خمسة أثمانية مائتين وهو ماية و خمسة و عشرون
وهذ الشيء و ذلك وصية الذي اوصى له بالجاربة
* فالسفي عبد الله يقيمته ماية درهم و وجب لرجل جارية
قيمتها خمس ماية درهم و عقرا ماية درهم فتوها الموجب له
واوصي الواهب لرجل بثلاث ماله فقياسه في قول أبي
حنيفة أنه لا يضرب صاحب الجارية باكثر من الثلاث
فيكون الثلاث بينهما نصفين * و قيساه ان تجعل قيمة
الجاربة خمس ماية درهم الوصية من ذلك شيء فنصار في
ايدي الوصية من ذلك خمس ماية درهم غير شيء واحد
و العقر ماية غير خمس شيء فصار في اديهم ستماية غير
شيء و خمس شيء وأوصي لرجل بثلاث ماله وهو مثل
وصية صاحب الجارية وهو شيء فيقب في ايدي الوصية
ستماية غير شيئين و خمس شيء و ذلك مثل واضاهم
جميعا قيمة العبد والشيئين الموصى بهما فنصف ذلك
يعدل وضاهم وهو ثلاثماية غير شيء و عضو شيء فاجبر
ذلك شيء و عضو شيء فتكون ثلاثماية يعدل ثلاثة اشياء
و عضو شيء و ماية درهم فاطر ماية جارية فيقب فيميتان
يعدل ثلاثة اشياء و عضو شيء فقابل به فالشيء من ذلك
سبعة وعشرين جزءًا من شيء فقابل به وتخطه الي شيء واحد وذلك أن تنقص منه سبعة أجزاء من أربعة وثلثين جزءًا من يكون شيء الواحد يعدل مائتي درهم وعشرة دراهم وخمسة أجزاء من سبعة عشر جزءًا من درهم وهو الوصية.

فان اعتق عبدًا له في مرضه قيمته مائة درهم وهب لرجل جارية قيمتها خمسمائة درهم وعقارها مائة درهم فوالتها الموهب له وقول أبي حنيفة أن العتق أولي فتبنا به قباحة أن تجعل قيمة الجارية خمسمائة درهم في قوله وقيمة العبد مائة درهم ونجعل وصية صاحب الجارية شيئاً آخر فقد اضمني صوتي العبد وقيمتة مائة درهم وأوصي الموهب له بشيء وزد العقار مائة درهم غير خمس شيء فصار في إدي الورثة ستمائة درهم غير شيء وخمس شيء وهو مثلما المائة الدرهم والشيء نصف ذلك مثل وصيتهما وهو ثلاثمائة خمسة أخماس شيء، فاجب الثلاثمائة بثلثة أخماس شيء وزد مثلها على الشيء، فتكون ذلك ثلاثمائة درهم يعدل شيئاً وثلثة أخماس شيء ومائة درهم فاطرح من الثلاثمائة مائة خمسة فبقي مما أبداهم يعدل شيئاً وثلثة أخماس شيء.
وستن درهما وثلاثان وثلاث شئ ولابنتة مثل ذلك تضمنه إلي ما تركته وهو ثلاثمائة درهم فيكون ثلاثمائة وستن درهما وثلاثي درهم وثلاث شئ وقد أستبت البلك مالها وهو مائة درهم واثنان وعشرون درهما وتسع درهم وتسعا شئ وتبقي مائتان ورابعة واربعون واربعة انساع درهم وتسعا شئ ليل من ذلك البلك واحد وثمانون درهما واربعة انساع وثلاث تسع شئ ورجع ما بقي إلي السيد وهو مائة واثنان وستن درهما وثمانية انساع وثلاث تسع درهم وتسعا شئ وثلاث تسع شئ ميزانا له فنصحه فنصب في أيدي ورثة السيد خمسة وتسعة وعشرون درهما وسبعة عشر جزء من سبعة وعشرين جزءا من درهم غير اربعة انساع شئ وثلاث تسع شئ وثلاث مثل الوصية التي هي شئ فنصب ذلك مائتان واربعة وستن درهما واثنان وعشرون جزءا من سبعة وعشرين جزءا من درهم غير سبعة اجزاء من سبعة وعشرين جزءا من درهم فاجبر ذلك بالسعة الأجزاء وتزيد عليها الشئ فتكون ذلك مائتين واربعة وستين درهما واثنين وعشرين جزءا من سبعة وعشرين جزءا من درهم يعدل شيئا وسبعة أجزاء من
السعة ثلثمئة غير شيء فيبقى شيء للبندت نصفه وللسيد نصفه فضيف حصة البندت وهي نصف شيء التركب، وهي ثلثمئة فيكون ثلثمئة درهم ونصف شيء للزوج من ذلك النصف ويرفع إلى السيد النصف وهو مائة وخمسون وربع شيء فصار جميع ما في يد السيد اربعمائة وخمسين غير ربع شيء، فذلك مثل الوضعية نصف ذلك مثل الوضعية وهو مائتان وخمسة وعشرون درهما غير ربع شيء يعدل شيئا فاجبر ذلك بثم شيء وزده على الشيء فتكون مائتين وخمسة وعشرين درهما يعدل شيئا وخمس شيء فقابل بذلك الحال، الواحد مئانية اتساع مائتين وخمسة وعشرين وذلك مائتي درهم.

فان اعتق عبدا له في مرضه قيمته تلثمئة درهم فماتت العبد، وترك خمسمئة درهم وترك بنتا وأوصي بثلاث ماله ثمن مات البندت وتركت إما وأوصت بثلث مالها وتركت تلثمئة درهم فقياسه أن ترفع من تركة العبد السعة، وهي تلثمئة درهم غير شيء فيبقى مائتا درهم و شيء وقد اوصى بثلث ماله وهو ستة وستون درهما وثلثان وثلاث شيء ويرفع إلي السيد بميزانه ستة
غير ثلث شيء، ثم تقضي من ذلك الدين المالي وهو
ثلثماية درهم فيبقي سبعماية درهم غير ثلث شيء، وهو
مثلا وصية العبد وهي شيء. نصف ذلك ثلثماية وخمس
غير ندس شيء، يعدل شيئاً ناجباً ذلك بنسد شيء يكون
ثلثماية وخمسين. يعدل شيئاً ونسد شيء يكون الشيء
ستة أسابع الكلماية والخمسين وهو ثلثماية درهم وذلك
الوصية فتجمع تركه العبد وما استثناه من ذلك.
وثلثماية وخمسون درهماً فتعزل من ذلك الدين مالي
درهم ثم تعزل السبعاية وهي قيمة الرقبة غير الوصية مالي
درهم فيبقي الفن وتسعماية درهم وخمسون درهماً لام
من ذلك الثلاث ستمائة درهم وخمسون درهماً فالتمله
والن الدين وهو مايتن درهم من تركه العبد الموجودة وهي
الف وسبعماية وخمسون درهماً فيبقي تسعماية درهم
تقضي منها دين المالي ثلثماية ويبقي ستمائة درهم وذاك
مثال الوصية.*

فان اعتن عبدًا له في مرضا قيمته ثلثماية درهم ثم
مات العبد وترك بنتاً وترك ثلثماية درهم ثم ماتت
البنت وترك زوجاً وترك ثلثماية درهم ثم ماتت
السيد فقيمة أن تجعل تركه العبد ثلثماية درهم وتجعل
العبد وما تجبل منه المالي لذك الف وخمسة درهم
فطعن من ذلك السمعية وهي مائتان وعشرون درهما
فيقي الف ومائتان وثمانون درهما للثاني النصف ستمية
واربعون درهما فتلقاه من تركه عبد وهي الف درهم
فيقي ستمية وستون درهما فتلقاه من ذلك دين المولي
مائتان درهم ويقي فييدي الورثة مائة وستون درهما
وذلك مثل الإوصية
فل أن اعتق عبد له في رضيه قيمته خمسة ستمية درهم
فتجمل منه ستمية درهم فاستهيلته وعلي المالي دين
ثلاثمئة درهم ثم مات عبد وترك له ووالده وترك
الناف وسبعمية وخمسمين درهما وعلي عبد دين مائتين
درهم فقياسه ان تجعل ترك عبد الفا وسبعمية وخمسمين
درهما الذي تجبل المالي وهو ستمية درهم فذلك الفان
وثلاثمئة وخمسمين درهما فتعزل منه الدين مائتي درهم
وتعزل منه السمعية خمسة درهم غير شيء والوصية
شيء فيقي الف وسبعمية وخمسمين درهما وشيء للمن
ذاك ثلاث خمسة وخمسمين وثلاث شئ فتلقاه هو والدي الذي هو مائتين درهم من تركه عبد
الموجودة وهي الف وسبعمية وخمسمين فيقي الف درهم.
ثلثمائة ومايامان استثلكما الملي وذلات خمسماية درهم
فيعطي الملي السعاية وهي مايامان وعشرون درهماً وبيقي
مايامان ومغانو للثأب النصف من ذلك ماية واربعون
درهماً فتلقية من تركه العبد وهي ثلاثماية فيقي في أيدي
الورثة ماية وستون درهماً وذلات مثلية وصية العبد التي
هي شيء
فان اعتق عبد الله في مرزة قيمته ثلاثماية درهم ود
تجل الملي منه خمسماية درهم ثم مات العبد قبل
موت الملي وترك ألف درهم وترك ابنه وعلي الملي
دين مايامان درهم قياسية ان تجل تركه العبد ألف درهم
فلالثماية التي استثلكها الملي السعاية من ذلك ثلاثماية
غير شيء فيقي ألف ومايامان وشيء والنصف من ذلك
لابنة العبد وهو ستمائة درهم ونصف شيء قياسية من
تركه العبد وهي ألف درهم فيقي اربعماية درهم غير
نصف شيء تقسي من ذلك دين الملي وهو مايامان درهم
فيقي مايامان غير نصف شيء يعدل مثلية الوصية
التي هي شيء وذلات شيئان فأجبر ذلك بوصف
شيء فيكون مايامان درهم يعدل سبنين ونصفاً فقابل به
فالمشي يعدل ثمانيين درهماً وهي الوصية فتجمع تركه
ونصف شيء، فيصير ثلثهما درهم يعدل حمسة أشياء ونصف شيء مقابل به فيصير الشيء الواحد ماية وسبعة والعشرين درهماً وثلثة أجزاء من أحد عشر سنتاً. وفان اعتنق عبداً له في مرضه قيمته ثلثهما درهم وقد تجعل المولي منه مايتاً درهم فاستقبلها ثم مات العبد قبل موته السيد وتزكى بيتاً وترك ثلثهما درهم قياسه أن تجعل تركتة العبد الثلثهما والمايتين اللتين استهلكهما المولي فذلك خمسماية درهماً فتعزل منها الساعية وهي ثلثهما غير شيء، لأن وصيته شيء، فيبيقي مايتاً درهماً وشيء للبيبة من ذلك النصف ماية درهماً ونصف شيء، ويترجع إليه وراثة السيد النصف بالمبرات وهو ماية درهماً ونصف شيء، في ادIMUM من الثلثهما والدرهم غير شيء، ماية درهماً غير شيء، لأن المائتين مستهلكان فيبيقي في ادIMUM بعد المائتين المستهلكين مايتاً درهماً غير نصف شيء، وفان ذلك يعدل وصية العبد مرتين فنصفيها ماية غير ربع شيء، يعدل وصية العبد، وهي شيء، فتجرز ذلك بربع شيء، فيكون ماية درهماً يعدل شيئاً وربع شيء، فالشيء من ذلك أربعة خمسات وهو ثمانون درهماً وهي الوعبة والسعادة مايتاً وعشرون درهماً فتجمع تركتة العبد، وهي
و خمسون درهماً غير شيئين وسدى شيئ وهو مثلاً
الوصتين جميعاً التين هما شيطان وثليتاً شئ فاضرب ذلك
فيكون شمالي مائة وخمسين درهماً يعدل سبعه اشياء
و نصفا قابلاً به فيكون الشيء الواحد يعدل مائة وثلة
عشر درهماً وثالث درهماً وذلك وصية العبد الذي قبته
ثلثامائة درهماً ووصية العبد الآخر مثل ذلك وثلث
وثلث مائة وثمانية وثمانون درهماً وثمانية أبسام درهماً
و سماياه ثلثامائة وأحد عشر درهماً وتسع درهماً
قان اعتق عددين له في مرضه قيمة كل واحد منهما
ثلثامائة درهماً ثم مات احدثهما وترك خمساية درهماً
وتزك بتنا وترك السيد ابنا فقيسه ان تجعل وصية كل
واحد مثناها شيئاً و سماياه ثلثامائة غير شيء وجعل
تركه البيت منهما خمساية درهماً وسماياه ثلثامائة غير
شيء ثقيقي ما ترك مائتان وشيء. فرجع الى مولاه
بالمراة مائة درهم ونصف شيء فيصير في ايدي ورثته
مولاه اربعامائة درهم غير نصف شيء. ويأخذون من
العبد الآخر سماياه ثلثامائة درهماً غير شيء فيصير في ايديهم
سبعمائة درهماً ونصف شيء. فنذل مثلاً وصيتهما التي
هي الشيطان وراشت اربعة اشياء فاجبر ذلك بشيء.
بقي في الائحة ويسعى الآخر في ماييس وثلاجة وثلاثين
درهم وثلاث
فان اعتق غدرين له في رضيه قيمة احدهما ثلاثية
درهم وقيمة الآخر خمسية درهم، فمات الذي قيمته
ثلثماية درهم وترك بتنا وترك السيد ابنا وترك العبد
اربعمئة درهم في كم يؤدي كل واحد منهما قياسه أن
تجعل وسية العبد الذي قيمته ثلاثية درهم فيها وساعته
ثلاثية غير شيء، وتجعل وسية العبد الذي قيمته
خمسية درهم فيها وثلي ثلي شيء، وساعته خمسية
درهم غير شيء، وثلي شيء، لأن قيمته مثل قيمة
الول وثلي ثليها، فإذا كان لذلك شيء كان لهذا مثله
وثلثي ثلثي، فمات الذي قيمته ثلاثية درهم، وترك
اربعمئة درهم تودي من ذلك السعاية ثلاثية غير شيء
في قبقي في إيدي وثليته مائة درهم؛ وشيء النصف من ذلك
لبنته، وهو خمسون درهماً ونصف شيء وما بقي لورثة
السيد، وهو خمسون درهماً ونصف شيء مضاف إلي
ثلاثية غير شيء، فيكون ثلاثية وخمسين غير نصف
شيء، وثباثوا من الآخر سعاية، وهو خمسية درهم
غير شيء، وثلي شيء، فيصير في إيديهما الثلاثية
عشرة درهما وتسعة شيء فيصر في ايدي ورثة الولي
ثلثمئة وعشرون غير سبعة اتساع شيء يقضي من ذلك
دين الولي عشرون درهما فيقي ثلثمئة غير سبعة اتساع
شيء وذلك مثلما كان للعبد من الوصية التي هي
شيء وذللك شيئان تجبر الثلثمئة بسعة اتساع شيء
تزيد ذللك على الستين فيقي ثلثمئة يعدل شيئين
وسعة اتساع شيء الستين من ذلك نسعة أجزاء من
خمسة وعشرين فيكون ذللك على ثمانية وثمانية وذلک
كان للعبد
فكان اعتن البعض له في مرضه ولا مال له غيرها وقيمة
كل واحد منهما ثلثمئة درهم فتعجل الولي من اندهما
ثالث قيمته فاستهلكها ثم مات السيد فإماله تلك قيمة
الذي تعجل منه نمال السيد جميع قيمة الذي لم يتعجل
منه وثلث قيمة الذي تعجل منه وهو مائة درهم وذلک
اربع مائة درهم وثلث ذلك بيتهما مصفان وهو مائة
درهم وثلثه وثلثون درهما وثلث درهما لكل واحد منهما
ستون درهما وثلثا درهما فيسعي الذي تعجل منه
ثالث قيمة في ثلاثة وثلثين درهما وثلث لن له من
المائة نسوة وستين درهما وثلثي درهما وسبي xâyما
شيئًا وترك بنتا له من ذلك النصف وهو نصف شيء والموالي مثل ذلك فصار في ائدي ورثة الموالي للثمنية غير نصف شيء وهو مثل الوصية التي هي الشيء، وذلك شيئان فتجبر التلميذية بصف شيء وتزيد ذلك على الشئين فيكون ثلثماية يعدل شيئين ونصفا فالمشي من ذلك خمسة، وهو مائة وعشرون وهي الوصية والسعادة ماية وهما من فائتون.

فإن كان اعتقده في مملة وقيمة للثمنية درهم فماعات ترك اربعماية درهم وعليه دين عشرة دراهم وترك ابنتين واحدي لجل浠 للمال وعلي السيد دين عشرون درهما فقياس ذلك إن تجعل وصية العبد من ذلك شيئًا وسعاته ما بقي من قيمته وهو ثلثماية غير شيء فماعات العبد وترك اربعماية درهم ودعته من ذلك السعادة الي الموالي [سعاته] وهي ثلثماية غير شيء فبيقي في ائدي ورثة العبد ماية درهما وشيء فقتيمي من ذلك الدين وهو عشرة دراهم ويبيقي تسعون درهما وشيء ووضي من ذلك بثلث وهو ثلاث درهما وثلث شيء ويبيقي بعد ذلك لورثته ستون درهما وثلاث شيء للابنتين من ذلك الثلاثة اربعون درهما واربعة اتساع شيء والموالي.
الأغنيات إذا كان العبد مات قبل السيد فان كان العبد مات بعد السيد جعلت ثلثي قيمته وما سعي فيه العبد الآخر بين الابن والبنت للذكر مثل خط الأغنيات وما بقي من بعد ذلك [من تركه العبد] فهو للذكر دون الأغني لن النصف من ميراث العبد لابنه العبد والنصف بالولاية السيد وليس للابنة شيء. * وكذلك لو اعتن رجل
عبد له في مرض موهته ولا مال له غيره ثم مات العبد قبل السيد *
فان اعتن الرجل عبدا في مرضه ولا مال له غيره فان العبد يسعى في ثلثي قيمته * فان كان السيد قد تعجل منه ثلثي قيمته فاستهلها السيد ثم مات السيد فان العبد يسعى في ثلثي ما بقي * فان كان قد استوفي منه قيمته كلها فاستهلها فلا سبيل علي العبد لانه قد *
أتي جميع قيمته *
فان اعتن عبدا له في مرض موهته ثلاث مائة يرهم ولا مال له غيره ثم مات العبد وترك ثلثمئية درهم وترك بنا فقيده أن تجعل وصية العبد شيئا ويسبي فيما بقي من قيمته وهو ثلثمئية غير شيء فصار في يد الملك السعيئ وهي ثلثمئية غير شيء ثم مات العبد وترك
باب العنق في المرض

إذا اعتنق الرجل عبدين له في مرره وترك السيد ابنا وابناء ثم مات أحد عبدين وترك مال أكثر من قيمته وترك ابنة فاجعل ثقي قيمته وما سعي فيه العبد الآخر وميراث السيد منه بين اليدين والبنين للذكر مثل خط

درهم من ذلك وصية المرأة شيء فيبقى مائة درهم وعشرة درهم غير شيء ويسير في ايدي ورثة المرأة عشرون درهما وشيء وأوصي من ذلك بلهبه، وهو ستة درهم وثلثان وثقل شيء ويرجع الي ورثة الزوج من ذلك بالمرات نصف ما بقي وهو ستة درهم وثلثان وثلث شيء فيسير في ايدي ورثة الزوج مائة وستة عشر درهما وثلثان غير ثقل شيء وأوصي من ذلك بلهبه وهو شيء فيما قبله مائة وستة عشر درهما وثلثان غير شيء وثلثي شيء يعدل مثل الوصيلي وذلك اربعة اشياء فاجير ذلك فيكون مائة وستة عشر درهما وثلثي درهم يعدل خمسة اشياء وثلثي شيء فالشيء الواحد يعدل عشرين درهما وعشرة اجزاء من سبعة عشر جزءا من درهم وهي الوصية فاقلم ذلك

۱۰۱
كان تزوجها علي مئة درهم و مهر مثلها عشرة دراهم
وراري لرجل بثلاث ماله فقياس ذلك ان تعطي المرأة
مهرها وهو عشرة دراهم نهيك تسعون درهماً ثم تعطي من
ذلك وصيتك شيئاً ثم تعطي الورثه له بالثلث أيضاً
شيئاً لان الثلث بينهما نصفان لا تأخذ المرأة شيئاً الا اخذ
صاحب الثلث مثله تعطي صاحب الثلث أيضاً شيئاً
ثم يرجع الي ورثة الزوج ميراثهم من المرة خمسة دراهم
ونصف شيء نهيك في يدي ورثة الزوج خمسة وتسعون
الشيء ونصفاً وذلك يعدل اربعة اشياء فاجبر ذلك
شيء ونصف شيء نهيك خمسة وتسعون يعدل خمسة
أشياء ونصف ناجولا انصاناً فتكون احد عشر نصفاً والدرهم
انصافاً فتكون مئة وتسعين نصفاً يعدل احد عشر شيئاً
فالشيء الواحد يعدل سبعة عشر درهماً وثلاثة اجزاء
من
أحد عشر IRS درهم في الورثة
تانزوجها علي مئة درهم و مهر مثلها عشرة دراهم
ثم ماتت قبل الزوج و تركت عشرة دراهم واوصى بثلاث
مالها ثم ماتت الزوج و تركت مئة وعشرين درهماً واوصى
لرجل بثلاث ماله فقياس ان تعطي المرأة مهرها عشرة
دراهم نهيك في يدي ورثة الزوج مئة درهم و عشرة
إن المرأة بجوز لها بالوصية ثلث جميع ما ترك الزوج فلمّا وصيتها شيئان فاجبر الثلاثة والتسعين والثمانين شبيهًا وثانياً على الشيئين فيكون ثلثة وتسعين درهماً وثالثًا يعدل شيئين وثانياً شيء فاليالي الواحد من ذلك هو ثلثة إثمانه وهو يعدل ثلثة إثمان الثلثة والتسعين والثمانين وهو خمسة وثلوث درهماً

فإن كانت المسئلة على حالها وعلى مرأة دين عشرة دراهم وواست بثث مالها فقيس ذلك أن تعطي المرأة عشرة دراهم مهماً وبقي تسعون لائحة وصية فتجعل وصيتها شيئًا فقيض تسعون إلا شيئًا وصير في يد المرأة عشرة دراهم وشيء فنصس من ذلك ديها عشر دراهم فقيضي لها شيء وواست من ذلك شيئًا وهو ثلث شيء فسيبى ثلثين شيء يرجع اليا الزوج من ذلك بالميراث نصفه وهو ثلث شيء نصار في اليد ورثة الزوج تسعون درهماً إلا ثلتين شيء وثالتين مثل الوصية التي هي شيء فذلك شيوان فاجبر التسعين فثاني شيء وثانيًا عليه الشيئين فيكون تسعين درهماً يعدل شيئين وثانياً شيء فاليالي من ذلك ثلثة إثمانه وهو ثلثة وثلثون درهماً وثالثة أرباع درهماً وهي الوصية
ببساطة فعِّل السهم مائة وثلثين وثلاثين نيكو سهام الفريضة الفا وسبعمنة وأثنين وثلاثين سهما والبهم الواحد يعدل مائة وثلاثين وثلاثين ولكنه ظلما واثنين وألف وسبعين فتتالي الوصية مائتان وثلاثة

ويبقي للورثة الفف وسبعمنة وثمانية وعشرون

حساب الدور

باب منه في الزواج في المرف* زجل تزوج امرأة في مرض موعته علي مائة درهم ولا ساها له غيرها ومهما ماتت امرأة ووصت بثلث مالها ثم مات الزوج فقياسه ان ترفع من المائة ما يصح لها من المهر وهو عشرة دراهم ويبقى تسعون درهما لها منه وصية فتجعل وصيتها شيئا من ذلك فيبقى تسعون درهما غير شيء فصار في يدها عشرة دراهم وشيء ووصت بثلث مالها وهو ثلثة دراهم وثلث درهم وثلث شيء فبقى ستة دراهم وثلثان وثلثا شيء فرجع الي الزوج من ذلك ميزاته النصف وهو ثلثة دراهم وثلث درهم وثلث شيء فيصير في ايدي ورثة الزوج ثلاثة وتسعون درهما وثلث درهم الا ثلثي شيء وهو مثل وصية المرأة وهو شيء
فعم مالك وهو أن تزيد على السهام ثلاثة أخماسها فيكون
ما لا يعد بثني سهم وخمس سهم فالسهم الواحد خمسة
فيكون المال سه وثلتين ونصيب خمسة والسومة
واحدة

فان تركت أمه وأمرأته وأربعة أخوات واحدها لرجل
بتكملة النصف بنصيب أمرأته واخته إلا السعي ما يبقى
من السهم بعد التكميلة فقياس ذلك انك إذا طرحت
النصف من الثلاثة بقي عليك سدس وذلك ما استثنى
وهو نصيب المرأة والأخت وهو خمسة سهم فالذي يبقى
من السهم خمسة سهم إلا سدس المال والسبان اللذان
استثناهما سبعة خمسة سهم إلا سعي سدس سل فتكون
معك ستة سهم وثلثاء اسابع سهم إلا سدس سل وسبعي
سدس سل فتزيد علي ذلك ثلثي المال فتكون معك
تسعة عشر جزءا من اثنين واربعين جزءا من سل وسيلة
اهم وثلثاء اسابع سهم يعد ثلاثي عشر سهما فالى من
هذه السهام فيبقى تسعة عشر جزءا يعدل سه اسماء واربعية
اسبع اسماء فتتم مالك وهو أن تزيد عليه ضعفه وأربعة
اجزاء من تسعة عشر جزءا فيكون معك سل يعدل اربعة
عشر سهما وسبعين جزءا من مائة وثلثة وثلتين جزءا من
فاطرح منه ثلاثة الستين وخذ عليه ما بقي معك يعه
الأثمان فيكون معك خمسة أسداس مائة ومائة ونصف
هم يعد ثلاثة عشر يسا فالت الففثا عشرهم مهما
ونصفهم فيميقي أحد عشر هما ونصف يعد خمسة
اقداس مائة فكمل مالك وله ان تزيد عليه السهام
خمسة فيكون مالا يعد ثلاثة عشر هما واربعة اخماس
نافعل السهم خمسة فيكون المائة نسعة وستين ونصف
اربعة اسمهم

رجل مات وترك ابنه خمس بنات ووصي لرجل
بتكملا الخمس والسادس بنصيب الابن الربع ما يبقى
من الثلث بعد التكملا خذ ثلاث مال فالت خمس
المائة وسادسة [منه] الستين فيميقي معك سهمان ال
اربعة اجزاء من مائة وعشرين جزءا من مال ثم زد
علي الاستثناء وهو نصف اسمهم الا جزءا فيميقي معك
هما ونصف الأ خمسة اجزاء من مائة وعشرين جزءا من
مال فذر عليه ثلثي المائة فيكون خمسة وسبعين جزءا
من مائة وعشرين جزءا من مال وسبعين ونصفا يعد
سبعة اسمهم فالت سبعين ونصفا من سبعة فيميقي معك
خمسة وسبعين من مائة وعشرين يعدل اربعة اسمهم ونصفا
عشر سهما فيقي عشرة اسم وخمسا اسم يعدل ثلاثة أخماس مال فنق المال وهو ما يزيد على ما ملكت من السهم ثلاثما فيكون ملك مال يعدل سعة عشر سهما وثلاث سهم فاجعل السهم ثلاثا فيكون المال البنين وخمسين والسهم ثلاث والوصية الأولى سعة والثانية سعة.

فان كانت الفريضة على حالا واصط لرجل بتكلمة خمس المال بنصيب الام ولاخر بسدا ما بقي من المال فالسهم ثلاث عشر فخض مالا فالاق منه خمسة الستين ثم الم ستة ما بقي ملك فيبقي ثلاثا مال ومثل سهم يعدل ثلثا عشر سهما فالاق سهما وثلي سهم من ثماني عشر سهما فيبقي ثلاثا مال يعدل احد عشر سهما وثلثا فنتم المكل وهو ما يزيد على السهم نصفها فيكون ملك مال يعدل سعة عشر سهما فاجعل المال خمسة وثمانين والسهم خمسة والوصية الأولى سعة والثانية عشر وب الق خمسة وستون سهما للربة.

فان كانت الفريضة على حالا واصت لرجل بتكلمة ثلاث المال بنصيب الام الا تكلمة ربع ما بقي من المال بعد التكملة بنصيب بنت فالسهم ثلاث عشر سهما فخض مالا.
ولأخر بتكلفة الخمس بنصيب ابنة فاجاز ذلك الورثة فاتم
الفرصة فتخذهما من ثلثة عشر ثم خذ مالاً فالت منه ثلثة
الألفهم اسم نصيب الزوج ثم الق ربع السمهم نصيب
الإمام ثم الق خمسه إلا اسم نصيب البنت فيقي المال
ثلثة عشر جزء من ستين جزء ومة اسم يعدل ثلثة
عشر سهماً فاللق الستة من ثلثة عشر سهماً فيقي ثلثة
عشر جزءاً من ستين جزءاً من مال يعدل سبعة اسم
فكتل المالك وهو ان تضرب السبعة اسم في اربعة
وثمانية اجزاء من ثلثة عشر فيكون مكث مال يعدل
اثنين وتلثين سهماً واربعة اجزاء من ثلثة عشر فيكون المال
اربعاءة وعشرين *
فكان الفرضة على حالها وواست لرجل بتكلفة
ربع المال بنصيب الام وأخر بتكلفة خمس ما يقي
من المال بعد الوصية الولي بنصيب بنت فاتم السمهم
الفرصة فتخذهما من ثلثة عشر ثم خذ مالاً فالت منه
ريع الاسم اسم ثم الق خمسه ما بقي مكث من المال
الاسهم ثم انظر ما بقي من المال بعد السمهم فخذ ذلك
ثلثة اخماس مال وسهمين وثليثة اخماس اسم يعدل
ثلثة عشر سهماً فاللق سهماً وثلثة اخماس اسم من ثلثة
باب الكتمة

أم ماتت و تركت شهري بنت و ابنتها و زوجها وأوصت لزوج بتكتمة خمس المال بنصيب بنت ولآخر بتكملة ربع المال بنصيب الأم فقياس ذاك أن تقيم سهام الفريضة فيكون ثلث عشر سهما فتائفه مالا فلتقي منه خمسة

الأ سهما نصيب بنت وهي الوصية الأولى ثم تلقي منه اضافاً ربعاً الستة نصيب الأم وهي الوصية الثانية فميقاته

١٨١

١ عشريَّةٌ جزء من عشرين جزء من مال و ثلثة أسم

١٨٢

١٨٣

١٨٤

١٨٥

١٨٦

١٨٧

١٨٨

١٨٩

١٩٠

١٩١

١٩٢

١٩٣

١٩٤

١٩٥

١٩٦

١٩٧

١٩٨

١٩٩

٢٠٠

٢٠١

٢٠٢

٢٠٣

٢٠٤

٢٠٥

٢٠٦

٢٠٧

٢٠٨

٢٠٩

٢١٠

٢١١

٢١٢

٢١٣

٢١٤

٢١٥

٢١٦

٢١٧

٢١٨

٢١٩

٢٢٠

٢٢١

٢٢٢

٢٢٣

٢٢٤

٢٢٥

٢٢٦

٢٢٧

٢٢٨

٢٢٩

٢٣٠

٢٣١

٢٣٢

٢٣٣

٢٣٤

٢٣٥

٢٣٦

٢٣٧

٢٣٨

٢٣٩

٢٤٠

٢٤١

٢٤٢

٢٤٣

٢٤٤

٢٤٥

٢٤٦

٢٤٧

٢٤٨

٢٤٩

٢٥٠

٢٥١

٢٥٢

٢٥٣

٢٥٤

٢٥٥

٢٥٦

٢٥٧

٢٥٨

٢٥٩

٢٦٠

٢٦١

٢٦٢

٢٦٣

٢٦٤

٢٦٥

٢٦٦

٢٦٧

٢٦٨

٢٦٩

٢٧٠

٢٧١

٢٧٢

٢٧٣

٢٧٤

٢٧٥

٢٧٦

٢٧٧

٢٧٨

٢٧٩

٢٨٠

٢٨١

٢٨٢

٢٨٣

٢٨٤

٢٨٥

٢٨٦

٢٨٧

٢٨٨

٢٨٩

٢٩٠

٢٩١

٢٩٢

٢٩٣

٢٩٤

٢٩٥

٢٩٦

٢٩٧

٢٩٨

٢٩٩

٣٠٠
سهم ما من مائتين واربعين سهماً من مال واربعة خمسات نصيب ودرهم واربعة اخماس درهم فخذ الثالث وهو ثمانون فالقي منه اثني عشر واربعة اخماس نصيب ودرهما واربعة اخماس درهم ثم التق ربع ما بقي معك ودرهما فيقي معك من الثالث احد وخمسون الا ثلثة اخماس نصيب والا دهمين وسعة اجزاء اثنين جزءاً من درهم ثم التق من ذلك ثمناً المال وهو ثلثون فيقي احد وعشرون الا ثلثة اخماس نصيب والا دهمين وسعة اجزاء اثنين جزءاً من درهم وثلثاً المال يعدل ثمانية انسبا فاجب ذلك بما نقص وزده على الثمانية الأنسبا فيكون معك مائة واحد وثمانون سهماً من مال يعدل ثمانية انسبا وثلثة اخماس نصيب والا دهمين وسعة اجزاء اثنين جزءاً من درهم وكمثل مالك وذلك أن تزيد علي ما معك تسعة وخمسين من مائة واحد وثمانين فيكون النصيب ثلثمائة واثنين وستين والدرهم ثلثمائة واثنين وستين والمال خمسة ألف وماينتين وستة وخمسين والرصايا من الأربع ألف وماينتان واربعة وسمن الثلاث اربعمائة وتسعة وثمانين والاثني ستمائة وسعة وخمسون *
واربعة اخماس نصيب فيبقى خمسة غير اربعة اخماس نصيب، فتنقل ربع ذلك إياها للوصية ودرهما فيبقى معك سهمان، وثلثة ارباع سهم الا ثلثة اخماس نصيب ثم الربع من المال. وهو ثلثة فيبقى عليك بعد الثلاث ربع سهم وثلثة اخماس نصيب فارجع إلي الثلاثين وهما ستة عشر فالتين من ذلك ربع واحد وثلثة اخماس نصيب فيبقى من المال خمسة عشر سهما وثلثة ارباع سهم غير ثلثه اخماس نصيب

[يعدل شمالية انصب] فأجاب ذلك، بثقة اخماس نصيب وزدها على الارض وهي شماليه، فيكون خمسة عشر سهما وثلثة ارباع سهم يعدل شماليه انصب وثقة اخماس نصيب فاتسم ذلك عليه فاما بلغ فهو القسم وهو النصيب والمال اربعة وعشرون، ويكمن لكل بنت سهم، وماية وثلثة وأربعون جزءا من ماية وأثنتين وسبعين جزءا من سهم. * فان اردت ان تخرج السهام، صحيحة فخذ ربع المال فالتين منه نصيب فيبقى ربع المال الا نصيب، ثم الق منه درهما ثم الربع خمسا ما بقي من الربع وهو خمس ربع المال الا خمس نصيب والا خمس درهما والثانية فيبقى اربعة اخماس الربع الا اربعة اخماس نصيب والا درهما واربعة اخماس درهما فالوصية من الربع اثني عشر
ودرهم وثلثي درهم فكمل المالك وهو أن تزيد على الرزعة الانصبا والخمسة الاسداس والدرهم وثلاثي الدرهم جزءاً من سبعه عشر جزءاً من نصيب ودرهم وثلثي عشر جزءاً من سبعه عشر جزءاً من درهم فاجعل النصيب سبعه عشما ودرهم سبعه عشر فيكون المال مائة وسبعة عشر وإن أردت ان خرج الدرهم صحبة فاعمل به كما وصفت لك ان شاء الله تعالى.

فان تركت ثلثة بنيين وابنتين ورشي لرجل بتمثل نصيب بت وبدرهم وآخر بخمس ما بقي من الربع وبدرهم ولا آخر بيع ما بقي من الثالث بعد ذلك كله وبدرهم ولا آخر بتم جميع المال فاجاز ذلك الورثة فقيهه على أن تخرج الدرهم صاحباً وهو في هذا الوجه أحسن هو أن تأخذ ربع المال وتميهم [فاجعله] أستاً والمال اربعة وعشرين فالتق من الربع نصيب فيقيه ستة غير نصيب ثم الق درهما فيقيه خمسة غير نصيب فالنق خمسما يبقى فيقيه اربعة غير اربعة اخماس نصيب ثم الق درهما اخر فيقيه مكث ثلثة غير اربعة اخماس نصيب فقد علمت ان الوصية من الربع ثلثة واربعة اخماس نصيب ثم ارجع الي النصف وهو ثمانية فالنق منه ثلثة
فما بلغ فهو القسم وهو النصيب وهو ثلثة وجزء من أحد عشر مائة درهم والثلث سبعة ونصف

* فان تزكر اسعة بنين واثني لرجل مثل نصيب احد بنية ال ربع ما يبقي من الثلث بعد النصيب وبردهم ولأخير بثلث ما يبقي من الثلث وبردهم فان الوصية من الثلث فخذ ثلث مال فلق منه نصيبا فيقي ثلث الا نصيبا ثم زد علي ما معتك ربعه فتكون ثلثا وربع ثلث ال نصيبا وربع نصيب والق درهما فيقي ثلث وربع ثلث الا درهما والا نصيبا وربع نصيب ثم الق ثلث ما يبقي معتك من الوصية الثانية فيقي معتك من الثالث خمسة استم من ستة استم من ثلث مال الا ثلثي درهم والا خمسة اسدا را نصيب ثم الق درهما اخر فيقي معتك خمسة استم من ثمانية عشرهما من مال الا درهما وثلثي درهم والا خمسة اسدا را نصيب فزيد علي ذلك ثلثي المال فتكون معتك سبعة عشرهما من ثمانية عشرهما من مال الا درهما وثلثي درهم والا خمسة اسدا را نصيب بعد اسعد انصارا فاجر ذلك بما نقص وزن ملكه علي الانصة فيكون سبعة عشرهما من ثمانية عشر من مال يعدل اسعد انصارا وخمسة اسدا را نصيب

N
خمسة انصباً فاجبر ذلك بنصف نصيب وبرهم وثلثة
اربع درهم وزدها علي الانصباً فيكون معك خمسة اسدياس
مال تعدل خمسة انصباً ونصف نصيب ودرهم وثلثة
اربع درهم فكمل مالك وهو اتزيد علي الانصباً والدرهم
والثلثة الاربع مثل خمساً فتكون معك مال يعدل ستة
انصباً وثلثة اخماس نصيب ودرهمين وعشر درهم
فجعل النصيب عشرة ودرهم عشرة فيكون المال سبعة
وثمانين سهماً * وان اردت ان تختر الدهم درهبا
صيحا فخذ الثلث فاطرح منه نصيباً فيكون ثلثاً اللانصيبا
واجعل الثلاث سبعة ونصبا ثم التلق ثلاث ما معك وهو
ثلث الثلاث فيبقي معك ثلثاً اللانصيب الا ثلثي نصيب
وهو خمسة دراهم الا ثلثي نصيب فاللق واحداً بالدرهم
فيبقي معك اربعة دراهم الا ثلثي نصيب ثم التلق ربع
ما معك وهو سهم الا سدس نصيب واللق سهماً بالدرهم
فيبقي معك سهمان الا نصف نصيب فنذ ذلك علي ثلثي
المال وهو خمسة عشر فيكون سبعة عشر الا نصف نصيب
يعدل خمسة انصباً فاجبر ذلك بنصف نصيب وزدها علي
الخمسة فيكون سبعة عشرهما يعدل خمسة انصباً ونصفا
فاتسع سبعة [عشر] علي خمسة انصباً ونصف نصيب
وكرهما وجزء من احد عشر من درهم * فان ارتد
ابن خرج الدربم صحيحا فلا تكمل مالك ذلك الطرح
من احد عشر واحدا بالدرهم واقسم العشرة الباقية على
النصاب اربعة تنصابا وهي اربعة وثلثة ارباع نصيب فيكون
القسم اثنين وجزء من سبعة عشر اجزاء من درهم فاجعل
المال اثنين عشر والنصاب سهرين وجزءين من سبعة عشر
جزء وان ارتد ان خرج النصيب صحيحا فتتم مالك
واجبرة فيكون الدربم احد عشر من المال
* فان ترك خمسة بنين وواصي لرجل امثل نصيب احدهم
وبعث ما يبقى من الثلث وبدرهم وربع ما يبقى
بعد ذلك من الثلث وبدرهم فخذ ثلثا فالتقي منه نصيبا
فيقي ثلث النصابا ثم التق ما يبقى معك وهو ثلث
الثلث النصيب ثم التق مما يبقى درهما فيقي
معك ثلثا الثلث النصيب والا درهما ثم التق مما
معك ربع وهو سهم من ستة اشهر من الثلث الا سدس
نصيب والا ربع درهم ثم التق درهما اخر يبقى معك
نصف الثلث الا نصف نصيب والا درهما وثلثة ارباع
درهم فرز علي ذلك ثلثي المال فيكون خمسة اسداس
مال الا نصف نصيب والا درهما وثلثة ارباع درهم يعدل
تبة واربعون والسنة من الربع عشرة والمستثنى من النصيب الثاني ستة فانهم ذلك

باب الوصية بالدرهم

رجل مات وترك أربعة ناس ناسى لج لجل يحمل نصيب احدهم وبرع وما بقي من اللف وبرهم قياس ذلك ان تاخذ ثلث مال لتقي منه نصيب فيقي ثلث الإنصب ثم تلقي ربع ما يبقي معك وهو ربع ثلث الإنصب ورقيق اضما درهما فيقي معك ثلثة أرباع ثلث مال وهو ربع المال الثلاثة أرباع نصيب والا درهما فترنذ ذلك على ثلثا المال يكون معك احد عشر جزءا من إثني عشر من مال الثالثة أرباع نصيب والا درهما يعدل أربعة نصيبا فاجبر ذلك بثلثة أرباع نصيب وبرهم فيكون أحد عشر جزءا من إثني عشر من مال يعدل أربعة نصيبا وثلثة أرباع نصيب ودرهما فكل مالك وهو أن تزيد على الإنصب والدرهم جزءا من أحد عشر جزءا منها فيكون معك مال يعدل خمسة نصيبا وجزءين من أحد عشر جزءا من نصيب
والنصيب الآخر فان قياسه ان تلقي من ربع مال نصيباً فيجلي ربع غير نصيب ثم تلقي خمساً ما يبقي من الربع وهو نصف عشر المال الا خمس نصيب ثم ترجع الي الثلث فتلتقي منه نصف عشر المال واربعة اخماس نصيب ونصيباً اخر فيجلي ثلث النصيب عشر المال والا نصيباً واربعة اخماس نصيب فنوز علي ذاك ربع ما يبقي وهو الذي استثناه فاجعل الثالث ثمانيين فإذا رفعت نصف عشر المال بقي منه ثمانيية وستون النصيباً واربعة اخماس نصيب فنوز علي ذاك ربع وهو سبعة عشر سهماً الاربع ما تنقص من النصيباً فيكون ذاك خمسة ثمانيين الا نصيبين وربع نصيب فنوز ذاك علي ثلثي المال وهو عاية وستون فيكون معك مال وسدس ثمن مال الا نصيبين ورحا يعدل ستة انصباً فاجبه ذاك بما نقص منه وزده علي الالنصباً فيكون مالاً وسدس ثمن مال يعدل ثمانيية انصب وربع نصيب فارده ذاك الي مال واحد وهو ان تنقص من الالنصباً جزءاً من تسعة واربعين جزءاً من جميعها فيكون مال يعدل ثمانيية انصب واربعة اجزاء من تسعة واربعين جزءاً من نصيب فاجبه النصيب تسعة واربعين فيكون المال ثمانيية وستة وتسعين والنصيب
تاخذ أيضاً ربع مال فتلقى منه نصيب فيقي معك ربع مال غير نصيب ثم تلقى ثلث ما يبقى من الربع فيقي
ثلثاً ربع إلا ثلثي نصيب فتزيد ذلك علي ما يبقى من
الثلث فيكون ذلك ستة و عشرين جزءاً من ستين جزءاً
من مال غير نصيب وثانية و عشرين جزءاً من ستين
جزءاً من نصيب ثم رد علي ذلك ما بقي من المال بعد
اختذت منه الثلث والربع وهو بربع و سدس فيكون ذلك
سبعة عشر جزءاً من عشرين جزءاً من مال يعدل سعة
نصباً و سعة اجزاء من خمسة عشر جزءاً من نصيب
فتنم مالك وهو أن تزيد علي ما معك من الاصبا ثلثة
اجزاء من سعة عشر جزءاً فيكون معك مال يعدل ثانية
انصباً و رمية و عشرين جزءاً من مائة و ثلثة و خمسين جزءاً
من نصيب ناجب التصيب مائة و ثلثة و خمسين فيكون
المال الثامنة و ثلثيماة و أربعة و اربعين والرمية من الثلث بعد
التصيب سعة و خمسون و الرمية من الربع بعد التصيب

احذ و ستون

فان تركت ستة بنين و اوصي لجل بمثل نصيب ابن
وجهم ما يبقى من الربع و لجل آخر بمثل نصيب
ابن اخر إلا ربع ما يبقى من الثلث. بعد الوصيين الوصيين
وخمس نصيب ثم تلقي من ذلك نصيب بعد اخري فيقيثلث وخمس ثلث الأنصابين وخمس نصيب ثم تزيد علي ذلك ما استثنى فيكون ثلث اثنين الثامنات ثلث الأنصابين واربعه عشر جزءا من خمسة عشر جزءا من نصيب ثم تلقي من ذلك نصف سدس جميع المال فيقي سبعة وعشرون جزءا من ستين من مال إلا ما ينقص من الأنصابا فذا علي ذلك ثلثي المال واجبره بما نقص من الأنصابا وزدها علي الانصابا فيكون معك المال وسبعة اجزاء من ستين جزءا من مال يعدل ثمانية انسبا واربعه عشر جزءا من خمسة عشر جزءا من نصيب فارد ذلك الي مال واحد وهو ان تنقص مما معك سبعة اجزاء من سبعة وستين منه فتكون النصيب مايتين وواحدة وبيصر المال كله الفا وستمائة وثمانية فإن كانت الفرضة علي حاليا وأوصي بمثل نصيب بنت وخمس ما يبقى من الثلاث بعد النصيب ومثل نصيب اخري وثلث ما يبقى من الربع بعد نصيب واحد فقياس ذلك ان الوصيتيان من الربع ومن الثلاث فتاخذ ثلث مال فتلقى منه نصيبا فيقي ثلث مال الا نصيبا ثم تلقي خمس ما يبقى وهو خمس ثلث الا خمس نصيب فيميقي أربعة اخماس ثلث الا أربعة اخماس نصيب ثم
تسعة أجزاء من سعة وخمسين جزءًا فيقي مال يعدل ثمانية أنصاً وثلث وعشرين جزءًا من سعة وخمسين جزءًا من نصيب فالنصيب سعة وخمسين جزءًا وسماً. وتكون مهام الفريضة ارتعامة وخمسة وتسعين سهماً والخمسان من ذلك مائة وثمانية وتسعون سهماً فارفع من ذلك التصبيين مائة وثمانية عشر سهماً يقبي ثمانيون سهماً ترفع منه المستثنى وهو ربع الاثنينين وخمسة ستة وثلاطون سهماً فيقي للموصى له أثناى وثمانون سهماً ترفعها من مهام الفريضة وهي ارتعامة وخمسة وتسعين سهماً فيقي ارتعامة وثلاثة عشر سهماً بين سبعة أنصاً لكل بنت سعة.

وخمسون والابن مثل ذلك.

فان ترث إبنين وأبنين وأوصى لرجل يمثل نصيب بنت الأخم وسبعة ما يقبي من الثلاث بعد النصيب لآخر يمثل نصيب بنت أخري الابن ما يقبي من الثلاث بعد ذلك كله وأوصى لرجل آخر بنصف مدس جميع المال فان هذه الوصايا كلها من الثلاث فتأخذ ثلث مال فتلقى منه نصيب بنت فيقي ثلث مال لا نصيباً ثم تحذي علي ذلك ما استثنى وهو خمس الثلاث إلا خمس نصيب فيكون ذلك ثلثاً وخمس ثلث إلا نصباً
وخمسة وخمسين والخمسين من ذلك ثلثمائة واثنان
ثم ارفع النصيب من ذلك وهو اثنان وثمانون فيبقي مايتان وعشرون ثم ارفع من ذلك الربع والخمس تسعة وتسعين سهماً فتبقي ماية وأحد وعشرون فنذ عليها ثلثة اخماس المال وهو اربعمائة وثلثة وخمسون فيكون خمسماة واربعة وسبعين بين سبعة أسمهم لكل سم اثنان وثمانون وهو نصيب البند وللابن ضعف ذلك

فالنسبة الفريضة على حالها وأوحي لرجل يمثل نصيب الابن الا ربع وخمس ما يباقي من الخمسين بعد النصيب فلاوصية من الخمسين ترفع من ذلك نصيبين لن للابن سهماً فيبقي خمساً مال الا نصيبين ورد ما استبدا عليه وهو ربع الخمسين وخمساً الا تسعة اعشار نصيب فيكون خمس مال وتسعه اعشار الخمس الا نصيبين وتسعه اعشار نصيب فنذ علي ذلك ثلثة اخماس المال فيكون مالاً وتسعه اعشار خمس مال الا نصيبين وتسعه اعشار نصيب يعد سبعة انصباً فاجبر ذلك بنصيبين وتسعه اعشار نصيب وزدها على الاصباً فيكون مملك مال وتسعه اعشار خمس مال يعدل تسعه انصباً وتسعه اعشار نصيب فارد ذاك الي مال واحد وهو ان تنقص معاً ممكّن

M
حيث سبعة أتمم لكل سهم مائة وثمانية وثمانون سهماً وهو
نصيب البند والابن ضعف ذلك.
فآن كانت الفرصة على حالها وواصي على خمس ماله
بمثل نصيب البند وآخر بربع وخمس ما يبقى من
الخمسين بعد النصيب فقياس ذلك أن الرمية من
الخمسين تأخذ خمس مال تلقي من النصيب فيبقى
خمساً مال الأنصاب ثم تلقي منه ربع وخمس ما يبقى
وهو تسعة أجزاء من عشرين جزءًا من الخمسين إلا
ذلك من النصيب فيبقى خمس وعشر الخمس إلا أحد
عشر جزءًا من عشرين جزءًا من نصيب الذي عليه ثلاثة
احمس المال يكون ذلك أربعة أخماس وعشر خمس
مال إلا أحد عشر جزءًا من عشرين جزءًا من نصيب يعدل
سبعة أتمابا فاجبر ذلك بحاد عشر جزءًا من عشرين جزءًا
من نصيب وزدها على السبعة فيكون ذلك يعدل سبعة
انصبا واحد عشر جزءًا من عشرين جزءًا من نصيب فتقم
مالك وهو أن تزيد على كل ما معك تسعة أجزاء من
أحد وأرعين جزءًا فيكون ميك مال يعدل تسعة انصبا
وسبعة عشر جزءًا من أشهرين وثمانين جزءًا من نصيب
فاجعل النصيب أشهرين وثمانين جزءًا فيكون السهام سبعاً
نصيب ابنه فاطرح من الوصية الأخرى وهي خمسة وسدة
فبقي سبع وأربعة إجزاء من خمسة عشر جزءا من سبع
الأتسعة عشر جزءا من ثلاثين جزءا من نصيب فرد علي
فلك خمسة إسبيع المال الباقية فيكون ستة إسبيع مال
 وأربعة إجزاء من خمسة عشر من سبع المال الأتسعة عشر
جزءا من ثلاثين جزءا من نصيب يعدل سبعة انصاب فاجبها
بتسعة عشر جزءا وزدها علي السبعة الانصبا فيكون ستة إسبيع
مال وأربعة إجزاء من خمسة عشر جزءا من سبع مال
يعدل سبعة انصبا وتسعة عشر جزءا من ثلاثين جزءا من
نصيب فلكل مالك وهو ان تزيد علي كل ما معك
احد عشر جزءا من أربعة وتسعين جزءا فيكون معك
مال يعدل ثمانية انصبا وتسعة وتسعين جزءا من مالية
وثمانية وثمانين جزءا من نصيب فاجب المال كله الفا
وستمائة وثلاثة والنصيب مائة وثمانية وثمانين ثم خذ سبعي
المال وهو اربعمئة وثمانية وخمسون فاطرح منه النصيب
وهذ ماية وثمانية وثمانون ويبقي مائتان وسبعون فاطرح
خمس ذلك وسدها تسعه وتسعين سهما فيبقي مائة
وأحد وسبعون سهما فرد عليه خمسة إسبيع المال وهو الف
ومائة وخمسة وأربعون فككون الفا وثمانية وستة عشر سهما
ثم اردت إليه ما أستنني وهو خمس الثالث إلى الخمس نصيب فيكون ثلثاً وخمس ثلث وذلك خمسان النصيباً وخمس نصيب ثم زد ذلك علي ثلاث المال فيكون مال وخمس ثلاث مال النصيباً وخمس نصيب يعد اربعة نصيب فاجر المال بنصيب وخمس نصيب وزده علي الاربعة النصيباً فيكون مال وخمس ثلاث مال يعدل خمسة نصيباً وخمس نصيب فارد ذلك الي مال واحد وهو ان تنقص مما معك نصف صمته وهو جزؤ من ستة عشر فنصير معك مال يعدل اربعة نصيب وسبعة اثنان نصيب فاجعل المال تسعة وثلاثين والمال ثلاث عشر والنصيب ثمانية نصيب من الثلث خمسها واحد نفر عليه الواحد الذي استنن أنه من الوصية فنصب الوصية سبعة وبقي من الثلث ستة نفر عليها ثلثي المال وهو ستة وعشرون فهماً فيكون اثنين وثلثين علي اربعة بنين لكل ابن ثمانية

فان ترك ثلاث بنين وبنات واو فيد لجل من سبعي ماله يمثل نصيب ابنه والآخر خمس وسدن ما يبي في السبعين فالوصية في هذا الوجه من سبعي المال فخذ سبعي المال نطر من نصيب ابنه. نصيب بنين سعاً مال إلا
في هذا النوع وقياسه أن تأخذ ثلث مال نتائج
منه التضيب في طبق ثلث مال النسبة ثم تنقس منه
رباع ما يبقى من الثلث وهو ربع ثلث الأرباع
نسبة في طبق ربع مال الثلثة ارباع نسبة فرد عليه ثلثي المال
فيكون أحد عشر جزءًا من الثلث عشر جزءًا من مال
الثلثة ارباع نسبة يعدل اربعة نصيب. فانظر ذلك
بكل ارباع نسبة وزدها على الربعة النصيب يكون مفتوح
اثنين عشر جزءًا من الثلث عشر مال يعدل اربعة
نصيب وثلثة ارباع نصيب فكمل مالك وهو ان تزيد على الربعة
النصبه والثلثة الأرباع جزءًا من أحد عشر فيكون ذلك
خمسة نصيب وجزؤين من أحد عشر نصيب يعدل
مال فاقع النصيب أحد عشر ومال سبعة وخمسين
والثلث تسع عشر ترفع ذلك النصيب أحد عشر فيطيق
منه ثمانية للموصى له بالرباع اثنان وبيتي ستة مرودة علي
الثلتين وها ثمانية وثلثون فيكون اربعة واربعين بين اربعة
بين لكل ابن أحد عشر ديماً

فإن ترك اربعة بنين واوصي لرجل بمثل نصيب ابن
الخمس ما يبقى من الثلث بعد النصيب فبلاصية من
الثلث فخذ ثلثاً وأخرج منه نسبة فيبوقي ثلث الأنصبة
جزء من خمسة وتسعون وسبعة أجزاء من مئتي فتحة مايزة وتسعة أجزاء وتبعة الثلث عشر في ماية وتسعة أجزاء وتزيد على ذلك ثماني جزءاً فيكون الفا واربعمئة وسبعة وتسعين ونصب الزوج ثلثماً وسبعة وعشرون

فإن تكرَّك اختمان وامرأة وارعي لرجل، بل مثل نصيب

احتك الأثناء ما يبقى من المال بعد الوصية. فقياس ذلك أن تقيم القيمة من الني عشر مهما لكل احتك ثلث ما يبقى من المال بعد الوصية. فهذا مال الوصية فانت تعلم أن نصيب ما يبقى مع الوصية يعدل نصيب احتك نصتم ما يبقى هو نسم مال الأثناء وصية نغتم مال الأثناء وصية مع وصية يعدل ثلث مال وثلث وصية ثمانية فالمال كله يعدل ثلثة أثمان مال وثلث وصية وثمانية وصية وافترج من المال ثلثة أثمان، فيبقى خمسة أثمان المال تعدل ثلثة وصية وأثمان وخمسة وصية فالمال كله يعدل خمس وصية وأثمان واربعية الحماس وصية فالمال ثماني وعشرون ووصية خمسة ونصب ثمانية

وفي وجه آخر من الوصايا رجل مات وترك أربعة بنين وارعي لرجل، بل مثل نصيب أحد بنيه ولآخر بربع ما يبقى من الثالث فاعلم أن الوصية إنما هي من ثلث
احد وثلاثين منها وهي مائة واربعة واربعون جزءا فيكون ذلك ستمائة وأربعين فالتنت ثمنها وعشرا مائة واربعة وأربعين ومل تسبب الزوج وهو ثلث وتسعمون فيقي اربعمائة وثلاة للنож من ذاك ثلاثة وتسعمون ولال اثنان وستون ولفكل بنت مائة واربعة وعشرون

فان كانت الفريضة علي حالها وواست لرجل مثل نصيب الزوج الا تسع وعشر ما يبقي من المال بعد التنصيب فقياس ذلك ان تقييم سهام الفريضة فتخذه من ثلثة عشر سهما وولصة من جميع المال ثلثة اسمهم فيباقي مال الأثلة اسمهم ثم استثنيت تسع وعشر ما يبقي من المال فهو تسع مال وعشر الا تسع ثلثة اسمهم وعشرها وذالك تسع عشر جزءا من ثلاثين جزءا من سهم فيكون ذاك مالا وتسعا وعشر الا ثلثة اسم وتسعة عشر جزءا من ثلاثين من اسم يعدل ثلثة عشر سهما تابع مالك بثة اسمهم وتقية عشر جزءا من سهم فقده على الثلاثة عشر مالا فيكون مالا وتسعا وعشر يعدل ستة عشر سهما وتسعة عشر جزءا من ثلاثين جزءا من اسم فذاك الي مال واحد وهو ان تنقص من ذاك تسع عشر جزءا من مائة وتسعة أجزاء فيقي مال يعدل ثلاثة عشر سهما وثمانين
ثلثة عشر سهماً للام من ذلك سهماً وانت تعلم أن
الوصية سهماً تتسع جميع المال فبقي منه ثمانية انساع
المال الأستثناء بين الورثة فتتم مالك وتعاء أن يجعل
الثمانية الأنساع الأستثناء ثلثة عشر سهماً فتزيد علي ذلك
سهرين فتكون خمسة عشر سهماً يعدل ثمانية انساع مال
ثم تزيد علي ذلك ثمناً وعلي خمسة عشر ثمنها وهو سهم
وسبعة أثمان سهم لصاحب النساع من ذلك النساع وهو
سهم وسبعة أثمان سهم للآخر الوضعي له بعذب نصيب الأم
سهماً فيبيقي ثلثة عشر سهماً بين الورثة على سهامهم وثمن
من مياء وخمسة وثلاثين سهماً.

فإن اوصت بهم نصيب الزوج وبثم المال وعشرة
فاقت سهام الفريضة فتكون ثلثة عشر سهماً ثم زد عليها مثل
نصيب الزوج وهو ثلثة فيكون ستة عشر وذلك ما بقي
من المال بعد الثمن والعشر وهو تسعه أجزاء من أربعين
سهماً والذي يبقى من المال بعد الثمن والعشر احد وثليثون
جزء من اربعين جزءاً من المال وهو يعدل ستة عشر سهماً
فكمل مالك وهو أن تزيد عليه تسعه أجزاء من أحد
وثلثين جزءاً فاضرب ستة عشر في احد وثلثين منها فيكون
ذلك ارباعهما وستة وتسعين فنزعليها تسعه أجزاء من
نصيب ابن وثلثي ما بقي من المال فخذ ثلثا فطرح
منه اربعة نصيب ابن فيبقي ثلث مال إلا اربعة
إثنان نصيب ابن ثم البقية ما بقي من المال وهو
تسع مال إلا سبع نصيب وثلثي سبع نصيب فيبقي
تسع مال الأسبي نصيب وثلثي سبع نصيب فنز ذلك
علي ثلاث مال فيكون ثمانية أنساب مال إلا سبي نصيب
وثلثي سبع نصيب و ذلك ثمانية اجزاء من واحد
وعشرين جزءا من نصيب يعدل ثلثة أنساب فنجبر ذلك
فتكون ثمانية أنساب مال يعدل ثلثة أنساب وثمانية اجزاء
من واحد وعشرين جزءا من نصيب فنقم مال ذلك وهو أن
تزيد علي الثمانية الأنساب مثل ثمنها ومعل الثمنا مثل
ثمانها فيكون ملك مال يعدل ثلثة أنساب وخمسة واربعين
جزءا من ستة وخمسين جزءا من نصيب والنصيب ستة
و خمسون مال مائتان وثلث عشر سهما والوصية الأولى
اثنان وثمانون سهما والثانية عشر و بقي مائة وثمانية
و إثنان لكل ابن ستة وخمسون سهما.

و في وجه آخر من الوصايا * امرأة ماتت و تركت
ابنتها وامها وزوجها و واست لرجل ب مثل نصيب الأم ولاخ
بتسع جميع المال قياس ذلك تقيم سهام الفريضة فتكون

٧٣
لليون ثلثة كم كانت تكون سهامهم فتوجه ذلك سبعة فخذ فريضة يكون لخميسها سبع وسبعها خمس وذلك خمسة وثلثون فزد عليه سبعها وهو عشرة فيكون ذلك خمسة وأربعين للموصى له من ذلك عشرة وكل ابن اربعة عشرة والبنت سبعة.

فإن ترك أبها وثلثة بنين وبنتا ووصي لرجل مثل نصيب واحد بنية إلا مثل نصيب بنت أخرى لو كانت فاقم سهام الفريضة واجعلها شيئًا ينقسم بين هواء الورثة وبينهما لو كانت معهم ابنة أخرى فتخذها ثلاثماي وستة وثلاثين فنصيب ابنة لو كانت خمسة وثلثون ونصيب ابن ثمانين سهما وبينهما خمسة وأربعون وهي الوصية نذرها على ثلاثماي وستة وثلاثين فيكون ذلك ثلاثماي وأحدا وثمانين فذلك سهام المال.

فإن ترك ثلاثة بنين ووصي لرجل مثل نصيب واحد البنين إلا مثل نصيب ابنة لو كانت وبنفسيما بقي من الثلاث فقيس ذلك ان تقسم سهام الفريضة على شيء ينقسم بين هواء الورثة وبينهما لو كانت معهم ابنة أخرى فيكون ذلك واحدا وعشرين فلو كانت معهم بنت أخرى لكان لها ثلاثة ونصيب ابن سبعة فقد اوصي له باربعة اسماع.
فصل ما بين خمسة نصيب وبين ما نصيب من الثلث وهو ثمانيه وثلثون من مائة وخمسة وتسعين من نصيب
الابن بعد اخراج الثلاث لهما لأن الذي له من حصة الثلث
ثمانيه أجزاء من ثلث عشرة من الثلاث وهو اربعون والذي
اجاز له من خميسي نصيب ثمانيه وثلثون فذلك ثمانيه
وسبعون فيخذ منه خمسة وسنتون ثلث ماله لهما والذي
اجاز له حصة ثمانيه وثلثون فان اردت تصحيح سهام
الفريضة صحتها فكانت من مائتي ألف وتسعة عشر الفا
وثلثمائه وعشرين * 

وفي وجه اخر من الوضاية رجل مات وترك اربعة
بنين وأمرأة وأوصي لرجل بمثل نصيب أحد البنين ان
نصيب المرأة سالم سهام الفريضة وهي اثنان وثلثون سهما
للمرأة ثم اربعه ولكل ابن سبعة فان تعلم ان الذي
وصي له به ثلثة اساع نصيب ابن فزيد على الفريضة ثلثة
اساع نصيب ابن وهو ثلثه وهي الوصية فيكون ذلك خمسة
وثلثين للمرضي له ثلثة اسهم من خمسة وثلثين سهما فبقي
اثنان وثلثون بين الوصية على مهامهم *

فان تركت ابنين وبين واوصي لرجل بمثل نصيب ابن
ثالث لون كان فالفوجه في ذلك ان تنظر الي ابن لا كان
لهم فنكرب سهام الفريدة في ثلث عشر يصبه من ثلثة الاف وثامنة وعشرين

فان أجاز الابن الخمسين لصاحب الخمسين ولم يجز للأخر شيئاً واجازت الام الربع لصاحب الربع ولم يجز للأخر شيئاً ولم يجز الزوج لهما إلا الثلث فاعلم أن الثلث للرجلين جائز على جميع الورثة يضرب فيه صاحب الخمسين بثمانية اجزاء من ثلثة عشر جزء وصاحب الربع بخمسة اجزاء من ثلثة عشر فاتم الفريدةعلي ما ذكرت لك فيكون اثنين عشر للزوج الربع وللم السيد وللابن ما بقي وقيمه انك تعلم أن الزوج يخرج من يده ثلاثة حصته على كل حال فينيغوي ان يكون في يده ثلث اسموان الام يخرج من يدها الثلث لكل واحد بقدر حصته وهي قد اجازت لصاحب الربع من حامه حصته فصل ما بين الربع وحصته من نصيبها وهي تسع عشر جزء من مائة وستة وخمسين من جميع نصيبها فينيغوي ان يكون نصيبها مائة وثة وخمسين حصته من الثلث من نصيبها عشرون سهماً والذي اجازت له ربع حصته وهو تسعة وثلثين وتتخذ ثلث ما في يدها لهما وتسعة عشر نهما للذي اجازت له حامه ثم الابن قد اجاز لصاحب الخمسين
الفريضة، فتاخذها من اثني عشر سهماً للابن من ذلك سبعة
اسهم، وللزوج ثلثة اسهم والام سهماً * وانت تعلم
ان الزوج يجوز عليه الثلث فينبغي ان يكون في يده
مثل ما يخرج من حضته للوصايا وفي يده ثلثة للوصايا سهماً
وله سهماً * واما الابن الذي اجاز الوصيتان جميعاً
فينبغي ان يوخذ منه خمساً جميع ماله وريعة فيبقي في
يده سبعة اسهماً من عشرين سهماً والذي له كل عشرون
سهماً * واما الام فينبغي ان يبقي في يدها مثل ما
يخرج من يدها وهو واحد وجميع ما كان لها اثنان
فخذملاً يكون لريعة ثلاث ولسدسه نصف ويكون ما يبقي
يتسق بين عشرين فذلك مايتان وأربعون * للام من
ذلك السدس وهو اربعون وصية من ذلك عشرون وليا
عشرون * وللزوج من ذلك الرابع ستون الوصية من
ذلك عشرون وله اربعون * ويبقي ماية وأربعون للابن
الوصية من ذلك خمسان وريعة وهو واحد وتسعون ويبقي
تسعه وأربعون فجميع الوصية ماية واحد وثلاثون بين
الرجلين الوصي لبما اصحاب الخمس من ذلك ثمانية
اجزاء من ثلثة عشر جزءاً ولصاحب الرابع خمسة اجزاء
من ثلثة عشر جزءاً فان اردت تصحيح سهام الرجلين الوصي
فتخذهما من عشرين فخذ مالاً فالتقي ثمته وسبعه فيبقى مال
الاثمنا وسبعا فتم مالك وهو أن تعدي عليه خمسة عشر
جزء من أحد وارعين جزءاً فاضرب سهام الفريضة وهي
عشرون في أحد وارعين فيكون ثماني مائة وعشرين فتميد
علي ذلك خمسة عشر جزءاً من أحد وارعين وهو ثلاثمائة
جزء فنصبر ذلك كله ألفاً وثمانية وعشرين سهماً للمعاصي
لكن ذلك بالثنم والسبع سبع ذلك وثمه وهو ثلاثمائة
السبع مائة وستون والثمان مائة وارعون فيبقى مائة
عشرون سهماً بين الوثيرة علي سهامهم *

باب الأخرى الوصايا

وهو إذا لم يجز بعض الوثيرة واجاز بعضهم والوصية أكثر
من الثالث * أعلم أن الحكم في ذلك أن من الإجاز
من الوثيرة أكثر من الثالث من الوصية فذلك داخل عليه
في حصة ومن لم يجز فالثالث جائز عليه علي كل حال. *
مثال ذلك امرأة ماتت وتركت زوجها وايتها وامها
واوصت لرجل خمس متالي وآخر برفع مالها فاجاز ابن
الوصيين جميعاً وأجازت الم النصف لهما ولم يجز الزوج
شيئاً من ذلك إلا الاثنين فقياس ذلك أن تقيم سهماً
جزء من شيء يعدل ثلاثة دراهم فأخذتي إليهسائل

وزيد مثل ذلك على ثلاثة دراهم وهو درهم وجزء من أحد عشر جزء فتكون أربعة دراهم وجزءا من أحد عشر جزءا من درهم يعدل شيئا وهو الذي استخرج من الدين.

باب آخر من الوصايا

رجل مات وتكر أمه وامرأته وأخاه وأختيه لبيته

وامرأته ووصي لرجل تسبع ماله فان قياس ذلك ان تقيم

في بعيد فاتخذها من ثمانية وأربعين سهما فانت تعلم أن كل

مال نقصت تسعة بقيت ثمانية انساع وأن الذي نزعه

مثل ثم ما أبنت فتزيد على الثمانية الأنساع ثمها

وعلى الثمانية والأربعين مثل ثمها ليتم المال وهو سنة

فيكون ذلك أربعة وخمسين للمرعي له بالتسعة من ذلك

سنة وهو تسعة مال وما بقي هو ثمانية وأربعون

بين الوصية على سهامهم.

فإن قال امرأة هلقت وتركت زوجها وابنها وثلث

بنات وأوصت لرجل بثمان مالها وسبع فاتم سهام الفريضة
خمسة مالاً وهو درهمان وخمس شيًّة فيبقي ثمانية
درهم ورابعة أخماس شيء ثم تعزل الدهم الذي اوصي
به فيبقي سبعة درهم ورابعة أخماس شيء فنقسمه بين
البنين فتكون لكل واحد ثلثة درهم ونصف درهم وخمسا
شيء [وهو يعدل الشيء فقابل به نفقي خمسة شيء] من
شيء فيبقي ثلث أخماس شيء تعدل ثلثة درهم ونصف
فكمل الشيء وهو ان تزيد عليه مثل ثلثية وتزيد علي
الثلثة والنصف مثل ثلثية وهو درهمان وثلاث فيكون خمسة
وخمسة اسداس وهو الشيء الذي استخرج من الدين.

فإن ترك ثلاثة بنين واويج خمسة مالاً الا درهماً وترك
عشرة درهم عيناً وعشرة درهم ديناً على احد البنين فإن
قياسه ان تجعل المستخرج من الدين شيئاً فنتزيله على
العشرة فتكون عشراً وشيئاً فتعزل خمسة للفصا والودرهمان
وخمس شيًّة فيبقي ثمانية درهم ورابعة أخماس شيء.
ثم تستعي درهماً لأنه قال الا درهماً فتكون تسعة درهماً
ورابعة أخماس شيء فنقسم ذلك بين البنين فتكون لكل
بنين ثلاثة درهم وخمس شيء وثلث خمس شيء فتكون
ذلك يعدل شيئاً فنفقي خمس شيء وثلث خمس
شيء من شيء فيبقي أحد عشر جزءاً من خمسة عشر
كتاب الوصايا

باب من ذلك في العين والدين

رجل مات وترك ابنين وأوصي بذلك ماله لرجل آخر وترك عشرة دراهم عينا وعشرة دراهم دينا على أحد الابنين قياسه ان يجعل المستخرج من الدين شيئا فترده على العين وهو عشرة دراهم فيكون عشرة وشيئا ثم تعزل ثلثها لأنه أوصى بذلك ماله وهو ثلثة دراهم وثلث ثلث وثلث شيء فيبقي ستة دراهم وثلثان وثلثا شيء فتنقسم بين الابنين فنصيب كل ابن ثلثة دراهم وثلث درهم وثلث شيء فهو يعدل الشيء المستخرج مقابل به نتلقى ثلثا من شيء بذلك شيء فيبقي ثلثا شيء يعدل ثلاثة دراهم وثلثا فتحتاج أن تكمل الشيء [تزيد عليه مثل نصفه وتزيد على الثلثة والثلث مثل نصفها فيكون خمسة دراهم وهي الشيء] الذي استخرج من الدين

فان ترك ابنين وترك عشرة دراهم عينا وعشرة دراهم دينا على أحد الابنين وأوصي لرجل خمس ماله ودرهم قياسه ان يجعل ما يستخرج من الدين شيئا فترده على العين فتكون شيئا وعشرة دراهم فتعزل خمسا لأنه أوصي م
العمود وتكسرها ثمانية وأربعين ذراعًا وهو ضرب العمود
في نصف القاعدة وهو ستة فجعلنا أحد جوانب المرابعة شيئًا
فثنياه في مثله فصار مالا فحفظنا ثم علمنا أنه قد بقي
لنا مثلثان عن جنبي المرابعة ومثلثة أخرى فاما المثلثان
اللنان علي جنبي المرابعة فيما متساويان وعموداها واحد
وهما علي زاوية قائمة فكسرها أن تضرب شيئًا في ستة الا
نصف شيء فيكون ستة شيء الا نصف مال وهو تكسر
المثلثين جميعا اللتان هما علي جنبي المرابعة فاما تكسر
المثلث العليا فهو أن تضرب ثمانية غير شيء وهو العمود
في نصف شيء فيكون أربعة شيء الا نصف مال فجميع
ذلك هو تكسر المرابعة وتكسر الثلاث المثلثات وهو عشرة
أشياء. تعدل ثمانية وأربعين هو تكسر المثلث العظمى فالمثلث
الواحد من ذلك أربعة اذرب كاهسخم ذراع وهو
كل جانب من المرابعة. * وهذه صورتها *

![Diagram](image-url)
وهو عشرون ذراعا فبلغ ذلك مائة وستة اذرع وثلاثي ذراع فاردا ان ناقسه منه ما زدنا عليه حتي يخرج وهو واحد وثلاثي الذي هو ثالث تكسير الثني في اثنيين في عشرة وهو ثلث عشر وثالث وذلك تكسير ما زدنا عليه حتي انخرط فادا رفعنا ذلك من مائة وستة اذريع وثلاثي ذراع بقي ثلث وتسعم ذراعا وثالث وذلك تكسير العمود المخروط وهذه صورة. *

وكان المخروط مدوراً فائقاً من ضرب قطره في نفسه سبعه ونصف سبعه فما بقي هو تكسيره.

فان قيل ارض مثلهً من جانبيه عشرة اذرع عشرة اذرع والقاعدة اثنا عشر ذراعا في خوفنا ارض سبعة كم كل جانب من المرعة فقياس ذلك ان تعرف عمود المثلث وهو ان تضرب نصف القاعدة وهو ستة في مثله فيكون ستة وثلاثين فانقصها من احد الجانبيين الاثنين مئتين في مثله وهو ماية بقيت اربعة وستون فخذ جذرها ثمانيه وهو
الكتاب نمنها مدوره قطرا سبعة اذرع وحديث بها اثنان
وعشرين ذراعا فان تكسرها ان تضرب نصف الفطر وهو
ثالث ونصف في نصف الدور الذي يحيط بها وهو أحد عشر
فيكون ثمانية وثمانين ونصف وهو تكسرها فان احبئت
فضرب الفطر وهو سبعة في مثله فيكون سبعة واربعين فان قص
منها سبعة ونصف سبعة وهو عشرة ونصف فيبقي ثمانية
وثلة ونصف وهو التكسيرو هذه صورتها

فان قال عمود مخروط أسفله اربعة اذرع في اربعة اذرع
وارتفاعه عشرة اذرع وراسه ذراعان في ذراعين وقد كتا
بيتنا ان كل مخروط معدل الرأس فان ثلث تكسره أسفله
مصورا في عموده هو تكسره فلما صار هذا غير معدل اردنا
ان نعلم كم يرتفع حتى يكمل رأسه فتكون لا رأس له فعلمنا
ان هذه العشرة من الطول كلها كعد الاثنين من الاربعة
فالثاني نصف الاربعة اذا كان ذلك كذلك فالاعشرة نصف
الطول والطول كله عشرون ذراعا فلما عرفنا الطول اخذنا
ثلث تكسر الاسبفل وهو خمسة وثلث فنصبناه في الطول
وهو أثنتي عشر والعمود ابدا يقع على القاعدة علي زاويتين قائمتين ولذلك سمي عمودا لأنه مستو خارب العمود في نصف القاعدة وهو نصف، سيكون أربع وثمانين وذاك تكسيها وذالك صورتها *

والجنس الثالث منفرجة وهي التي لها زاوية منفرجة وهي ممثل من كل جانب عدد مختلف وهي من جانب ستة ومن جانب خمسة ومن جانب سعة فمعرفه تكسيها هذه من قبل عمودها ومستقطب جهرا ولا يقع مستقطب الجهة هذه المثلثة في خوفها إلا علي النصل المطول فاجعله قاعدة ولو جعلت أحد النصلين القصيرين قاعدة لوقع مستقطب جهرا خارجا وعلم مستقطب جهرا وعمودها علي مثل ما علمتته في الحادة وعلى ذلك النقياس وهذه صورتها *

وأما المدورات التي فرغنا من صفتها وتكسيرها في صدر
منها علي شيء مما يلي أي التسعين شئت فجعلنا
الشي مما يلي القاعدة عشر فنصبناه في مثله فصار مالا
ونقصناه من ثلاثة عشر في مثلها وهو وتسعة وتسعة وستين
فصار ذلك مالي وتسعة وستين الإ مالا فعملنا ان جذرها
هو العمود وقد بقي لنا من القاعدة اربعة عشر ا شيئا
نصبناه في مثله فصار مالي وستة وتسعین وعال السنة
وعشرين شيئا نقصناه من الخمسة عشر في مثلها فبقي
تسعة وعشرون دهمه وثمانية وعشرين شيئا الا مالا وجذرها
هو العمود فلما صار جذرها هذا هو العمود وجذر مالي
وتسعة وستين الإ مالا هو العمود أيضا علمنا انها متساويان
فقابل هبما وهو ان تلقي مالا بمال لن المالين ناتسان
فيه تسعه وعشرون وثمانية وعشرين شيئا يعد مالي
وتسعة وستين فالقتل تسعه وعشرين من مالي وتسعة
وستين فبيقي مالي واربعون يعد ثمانية وعشرين شيئا
فالذي الواحد خمسة وهو مستطع الحجر مما يلي الثلاث
عشر و تمام القاعدة مما يلي النصل الآخر فهو نسعة فذا
اردت ان تعرف العمود فانصرف هذه الخمسة فمثلها
وانقصها من النصل الذي يليها متروبا في مثله وهو ثلاثة
عشر فبيقي مالي واربعة واربعون فجذر ذلك هو العمود
ملغ الخمسة في مثلاً وهو خمسة وعشرون نبقي خمسة وسبعون فخذ جذر ذلك فهو العمود وقد صار ضلعًا على مثليثين قائمتين فان ارتت التكسير فاضرب جذر الخمسة والسبعين في نصف القاعدة وهو خمسة وذالك إن تضرب الخمسة في مثلاً حتي تكون جذر خمسة وسبعين في جذر خمسة وعشرين فاضرب خمسة وسبعين في خمسة وعشرين فيكون الناتج ثماني مائة خمسة وسبعين فخذ جذر ذاك وهو تكسيرها وهو ثلاثة واربعون وشيء قليل

وهذه صورتها *

وقد تكون من هذه الحادة الزوايا مختلفة الابلاغ فاعلم ان تكسيرها يعلم من قبل مستطع جبرها وعمودها وهي ان تكون مثلثة من جانب خمسة عشر ذراعا ومن جانب اربعة عشر ذراعا ومن جانب ثلاثة عشر ذراعا فانما ارتت علم مستطع جبرها فاجعل القاعدة أي الجوانب شئت فيعلاها اربعة عشر وهو مستطع الجبر مستطع جبرها يقع
منها ستة أذرع وضلع منها ثمانية أذرع والتفرق عشر فحساب ذلك ان تضرب ستة في اربعة فيكون اربعة وعشرين ذراعاً وهو تكسرها * وإن احبت ان تحسبا بالعمود فان عمودها لا يقع الاعلي للصلع الطول لأن الضلعين التصويرين عمودان فإن ارتت ذلك تضرب عمودها في نصف التامَّة فما كان فهو تكسرها وهذه صورتها

واما الجنس الثاني فالمثلثة المتساوية الالضع حادة الزوايا

من كل جانب عشرة أذرع فان تكسرها تعرف من قبل عمودها ومسقط حجرها وأعلم ان كل ضلعين متساويين من مثلثة تخرج منها عمود علي قاعدة فان مسقط حجر العمود يقع على زاوية قائمة ويقع على نصف التامَّة سواء إذا استوا الضبان فان اختلاف مسقط الحجر عن نصف التامَّة ولكن قد علمنا ان مسقط حجر هذه المثلثة علي اي اضلاعها جعلته لا يقع الاعلي عليها نصفه خمسة أذرع معرفة العمود ان تضرب الخمسة في مثلا وتضرب احد الضلعين في مثله وهو عشر فيكون مثلاً نتيجته صورتها


*
فيخرج إلى حساب المثلثات فاعلم ذلك وهذه صورة المثلثة المعينة.

وأما المثلثات فهي ثلاثة اجناس القائمة والمحاطة والمنفرجة * واما القائمة فهي مثلثة إذا ضربت ضلعها الاقترنين كل واحد منها في نفسه ثم جمعتيمهما [كان مجموع ذلك مثل الذي يكون من ضرب الصلع الطول في نفسه * واما المحاطة فهي مثلثة إذا ضربت ضلعها الاقترنين كل واحد منها في نفسه ثم جمعتيمهما] كانا أكثر من الصلع الطول مضروبًا في نفسه * واما المنفرجة فهي كل مثلثة إذا ضربت ضلعيها الاقترنين كل واحد منها في نفسه وجمعتيمهما كانا أقل من الصلع الطول مضروبًا في نفسه *

فاما القائمة الزوايا فهي التي لها عمودان وقطر وهي نصف مرحة فمفرعة تكسرها إن تضرب أحد الضلعين المحطرين بالزاوية القائمة في نصف الآخر فما بلغ ذاك فهو تكسرها * ومثال ذلك مثلثة قائمة الزاوية قلعة

١
وأما المعينة المستوية الاتجاع التي كل جانب منها خمسة اذرع ناحية تطيرها ثمانية، والآخر ستة اذرع فأعلم أن تكسرها إن تعبر التجرين أو احدهما فان عرفت التجرين جميعًا فان الذي يكون من ضرب احدهما في نصف الآخر هو تكسرها وذلك إن تنصره ثمانية في ثلاثة أو أربعة في ستة فيكون أربعية وعشرين ذراعًا وهو تكسرها فان عرفت قطرا واحدًا فقد علمت انها مثقال كل واحد منها ثلاثًا خمسة اذرع خمسة اذرع والضلع الثالث هو قطرا كافهًا فاحسبهما على حساب المثلثات وهذه صورتها *

وأما المشبعة بالمعينة فعلي مثل المعينة *
وأما سائر المربعات فانما تعبر تكسرها من قبل التجر
أعلم أن المراعات خمسة أجناس فمنها مستوى الإضلاع قائمة الزوايا والثانية قائمة الزوايا مختلفة الإضلاع طوليا أكثر من عرضاً والثالثة تسمى المعيقة وهي التي استوت اضلاعها واختلفت زواياها والرابعة المشهية بالمعينة وهي التي طولياً عرضاً مختلفان وزواياها مختلفة غير ان الطولين مستويان، والعريض مستويان أيضاً والخامسة المختلفة الإضلاع والزوايا فإن كان من المراعات مستويات الإضلاع قائمة الزوايا أو مختلفة الإضلاع قائمة الزوايا فإن تكسيرها أن تخرب الطول في العرض فأما لغ في التكسر، ومثال ذلك أرض مربعة من كل جانب خمسة أذرع تكسيرها خمسة وعشرون درعاً وهو هذه صورتها.

والثانية أرض مربعة طولياً ثمانية أذرع ثمانية أذرع والعريض ستة ستة فتكسيرها أن تخرب ستة في ثمانية فيكون ثمانية وأربعين درعاً وذلك تكسيرها وهذه صورتها.
النقطة خ طلا يقطع سطح ا ك بتصنيف فحدث من السطح مثليين وهما مثلهما اطأ و دك ف قد تبين لنا ان نصف اب و أه مثله وهو نصف اج وتورهما خط طه على زاوية قائمة وكذلك خرج خطوطه من ط الي روس ير الي ج و س ج الي ى فحدث من جميع المبعة ثماني مثلثات متساويات وقد تبين لنا ان أربع منها نصف السطح الأعظم الذي هو اد وقد تبين لنا ان خط اط في نفسه تكسر مثلثين و اه تكسر مثلثين بمثلهما ف يكون جميع ذلك تكسر أربع مثلثات و ضلع اط في نفسه ابنا تكسر أربع مثلثات اخر و قد تبين لنا ان الذي يكون من ضرب اط في نفسه و اه في نفسه مجموعين مثل الذي يكون من ضرب طه في نفسه وذلك ما اردا ان نبين

وهذه صرحة *
حفظت أن كانت القوس أثقل من نصف مدور أو زده عليه أن كانت القوس أكثر من نصف مدور فما بلغ بعد الزيدات أو النقصان فهو تكسر القوس.

وكل جسم مربع فإن ضرب النصف الطول في العرض ثم في العميق هو التكسر. فان كان على غير تربع وكان مدوراً أو مثلثاً أو غير ذلك إلا أن عمقه على الاستواء والموازاة فإن مساحة ذلك فإن تمسح سطه فتعرف تكسره فما كان ضرره في العمق وهو التكسر.

وأما المخروط من المثلث والربع والمدور فان الذي يكون من ضرب ثلاث مساحة استهل في عموده هو تكسره.

واعلم أن كل مثلث قائم النزاوية فان الذي يكون من ضرب الضعفين اللائرين كل واحد منها في نفسه جمعيهم. مثل الذي يكون من ضرب الصعوب الأطول في نفسه.

وبهذا ذلك إذا نجعل سطحاً مربعاً متساوي الاتصال والزوايا أب جد ثم نقطع نثلب اج بنصفين على نقطة ثم نخرج في رئ نقطع نثلب أب بنصفين على نقطة ثم نخرج في نقطة ج فصار نثلب أب جد أربعة سطوح متساوية الاتصال والزوايا ومساحة وهي سطح أب وسطح جد وسطح ب جد ثم نخرج من نقطة ه.
من المثلثات والمراعات والمثلعتات وما فوق ذلك فان ذكرت
نصف ما يحيط بها في نصف قطر اوسع دائرة يقع فيها
تكريرها **و كل مدوره فإن قترها مضروبًا في نفسه
منقوصًا منه سبع و نصف سبع هو تكريرها وهو موافق
للباب الأول **

وكل قطعة من مدوره مشبة بقوس فلا بد ان يكون
مثل نصف مدوره أو أقل من نصف مدوره أو أكثر من
نصف مدوره والدليل علي ذلك أن سهم القوس اذا كان
مثل نصف الوتر فهي نصف مدوره سواء وإذا كان أقل من
نصف الوتر فهي أقل من نصف مدوره وإذا كان السهم
أكثر من نصف الوتر فهي أكثر من نصف مدوره **

وإذا اردت ان تعرف من اي دائرة هي غازب نصف الوتر في
مثله واقسمه على السهم وزد ما خرج عليه السهم فما بلغ
في قتر المدوره التي تلك القوس منها **

فإن اردت ان
تعرف تكير القوس غازب نصف قطار المدوره في نصف
القوس واحظ ما خرج ثم انتص سهم القوس من نصف قطار
المدوره ان كانت القوس اقل من نصف مدوره وإن كانت
أكثر من نصف مدوره فانقص نصف قطار المدوره من سهم
القوس ثم اضرب ما بقي في نصف وتر القوس وانتصه بما
مثل ربع السطح الذي هو من كل جانب درع وكذلك ثلث في نصف وربع في ربع وخمس في خمس وثلثان في نصف أو أقل من ذلك أو أكثر فعلي حسابه * وكل سطح مربع متساوي الاضلاع فان واحد اضلاعه في واحد جذره و في اثنين جذران صفر ذلك السطح أو كثر * 
و كل مثلث متساوي الاضلاع فان ضربات العمود و نصف القاعدة التي يقع عليها العمود هو تكسيز ذلك المثلث * 
و كل معينة متساوية الاضلاع فان ضربات أحد القطرین في نصف الآخر هو تكسيزها * 
و كل مدوره فان ضربات القطر في ثلثة وسبع هو الدور الذي ي Sit بنا وهو اصلال بين الناس من غير اضطرار و لاهل الهندسة فيه قولان اخرين احدهما ان تضرب القطر في مخلة ثم في عشرة ثم تأخذ جذرها اجتمع فما كان فهو الدور * فالقول الثاني لاهل المجرم منهم وهو ان تضرب القطر في اثنين وستين الفا وهو خرج فهو الدور وكل ذلك تقسم ذلك على عشرتين الفا فما خرج فهو الدور و كل ذلك قريب بعضهم من بعض * والدور اذا قسمته على ثلثة وسبع يخرج القطر * وكل مدوره فان نصف القطر في نصف الدور هو التكسيز لان كل ذات اضلاع وزوايا متساوية
عمل بستة أيام كم نصيبه فقد علمت أن الستة الأيام هي خمس الشهر وأن الذي نصيبه من الدرهم بقدر ما عمل من الشهر وقياس ذلك أن قوله شهر هو ثلاثين يوما وهو المسعر وتقوله عشرة دراهم هو السعر وقوله ستة أيام هو المسمى وتقلبه كم نصيبه هو الشهر فاضرب السعر الذي هو عشرة في المسمى الذي هو سبأته وهو ستة فتكون ستين فاقسمه على التذئاب التي هي العدد الذاخر وهو المسعر فيكون ذلك درهماً وهو النعم وهذا ما يتعامل الناس بينهم من الصرف والكيل والوزن.

باب المساحة

علم أن معنى واحد في واحد اناها هي مساحة وعمده ذواع في ذراع * وكل سطح متوازي الأضلاع والزوايا يكون من كل جانب واحد فان السطح كله واحد * فإن كان من كل جانب اثنان فهو متوازي الأضلاع والزوايا فالسطح كله اربعة امتثال السطح الذي هو ذواع في ذراع * وكذلك ثلاثة في ثلاثة وما زاد على ذلك أو نقص وكذلك نصف في نصف ذواع وغير ذلك من الكسور فعلي هذا وكل سطح ذواع يكون من كل جانب نصف ذواع فهو...
لكث باريعة فقوله عشرة هو العدد المسعر وقوله بستة هو السعر.
وفقه كم لكث هو العدد الجهول المئات وقوله باريعة هو العدد الذي هو الثمن فالأعداد المسعر الذي هو العدد مبائن للعدد الذي هو الثمن وهو الربيعة فاضرب العشرة في الاربع وحما المتباينان الظاهران فيكون اربعين فاتسعهما على العدد الآخر الظاهر الذي هو السعر وهو ستة فيكون ستة وثلثين وهو العدد الجهول الذي هو في قول القائل كم وهو المئات ومبائنها السنة الذي هو السعر.
والوجه الثاني قول القائل عشرة بثمانية كم ثم اربعة وربما قال اربعة منها كم ثمها فالعشرة هي العدد المسعر وهو مبائن للعدد الذي هو الثمن الجهول الذي في قوله كم والثمانية هي العدد الذي هو السعر وهو مبائن للعدد الظاهر الذي هو المئات وهو اربعة فاضرب العددين الظاهرين المتباينين احدهما في الآخر وهو اربعة في ثمانية فيكون اثنين وثلثين واتسعهما على العدد الآخر الظاهر الذي هو المسعر وهو عشرة فيكون ثلاثة وخمسا وهو العدد الذي هو الثمن وهو مبائن للعشرة التي عليها قسمت وهكذا جميع معاملات الناس.
وقياسها إن شاء الله تعالى.
فان سأل سائل فقال اجبراهجه في الشهر عشرة درادم.
فإن قال مال تعزل ثلثة اجذار ثم تضرب ما بقي في مثلاً فيعود المال فقد علمت أن الذي بقي هو جذر إيبا والمال أربعة اجذار وهو ستة عشر

باب المعاملات

علم أن معاملات الناس كلها فن البيع والشراء والصرف والإجارة وغير ذلك على وجهين بارعة اعداد يلفظ بها السائل وهي السعر والسعر والسهم والسهم والчисل الذي هو المسرع المبائن للعدد الذي هو السهم والعدد الذي هو السع مبائن للعدد الذي هو السهم وهذه الاربعاء الاعداد ثلثة منها ابداً ظاهرة معلومة وواحد منها مجهول وهو الذي في قول القابل كم عنه يسأل السائل. والقياس في ذلك أن تنظري إلى الثلاثة الاعداد الظاهرة فلا بد أن يكون منها اثنان كل واحد منها مبائن لصاحبه تسترضب العددان الظاهرين المتتابعين كل واحد منها في صاحبه نما بلغ فاقمه على العدد الآخر الظاهر الذي مبائن مجهول مما خرج كث فهو العدد المجهول الذي يسأل عنه السائل فهو مبائن للعدد الذي قسمت عليه.

ومثال ذلك في وجه منه إذا قيل لكث عشرة بستة كم
إجمال الأول كله من قبل أن تلقى ثلثه في ثلثة إجذاره كان ونصفاً لن ثلثه في ثلثة إجذاره مال فهو كله في ثلثة إجذاره مال ونصف وهو كله في جذر واحد نصف مال فجذر المال نصف والمال ربع فنلتا المال مسند وثلثة إجذار المال درهم ونصف فمعنى ما ضربت سدما في درهم ونصف خرج ربعاً وهو المال.

فإن قال مال تعزل أربعة إجذاره ثم تأخذ ثلث ما بقي فيكون مثل الربيعة الإجذار والمال سايتان وستة وخمسون فقيسه انك تعلم ان ثلث ما بقي مثل الربيعة الإجذار وان بقي مثل ثلث عشر جذره فذر عليه الربيعة الإجذار فيكون ستة عشر جذراً وهو جذراً المال.

فإن قال مال عزلت جذره وزدت على جذره جذر ما بقي فكان درهمين فذاؤ جذر مال فجذر مال إلا جذراً يعدل درهمين فاطس منه جذر مال والثمن من الدرهمين جذر مال فيكون درهمين الإجذارا في مثله أربعة دراهم ومال إلا الربيعة الإجذار يعدل مال إلا جذراً فقابل به فيكون مالاً وأربعة دراهم يعدل مالاً وثلثة الإجذار فنلتا مالاً جمال فرمى ثلثه إجذار تعدل أربعة دراهم فألجذر يعدل درهماً وثلثاً وهو جذراً المال والمالم درهم وسبعة اثناع درهماً.

*
مال وسدن جذر مقصوم على درهم يعدل درهما فكم الامل الذي معك وهو أن تصرفه في ستة فيكون معك المال وجدُر فاضرب الدراهم في ستة فيكون ستة دراهم فيكون المال وجدرا يعدل ستة دراهم فنصف الجذر وضربه في مثله فيكون ربعا أخذه على السعة وخذ جذر ما اجتمع فانقص منه نصف الجذر الذي كنت ضربته في مثله وهو نصف ما بقي فهو عدد الرجال الأولين وهم في هذه المسألة رجلان.*

لقد قال المال ضربته في ثلثه فكان خمسة قياسه انك اذا ضربته في مثله كان سعة و نصفا فتقول هو جذر سعة ونصف في ثلثي جذر سعة ونصف فاضرب ثلاثين في ثلاثين فيكون أربعة اتساع واربعة اتساع في سعة ونصف يكون ثلاثة وثلاثة فجذر ثلاثة وثلاثة هو ثلثا جذر سعة ونصف فاضرب ثلاثة وثلاثة في سعة ونصف فيكون خمسة وعشرين فجذرها خمسة.* فان قال المال تضربه في ثلثة اجذاره فيكون خمسة امثال المال الأول فكانه قال المال ضربته في جذره فكان مثل المال الأول وثليبه فجذر المال درهم وثلاثة والمال درهما وسعة اتساع.* فان قال المال ثلاثي ثليبه ثم تضرب الباقى في ثلثة اجذار المال الأول فيعود المال الأول وقياسه انك اذا ضربت
يُصرُّب شيئًا في ثلثي شيء يكون ثلثي مال يعدل خمسة
فإن كان نصفه وزن على النسبة مثل نصفها فيصير
معك مال يعدل سنة ونصفاً، فخذ جزءها وهو الشيء.

الذي تريد أن تصرِّبه في ثلثي فتكون خمسة

فإن قال مالان بينهما درهماً قسمت القليل على
الكثير قسمت نصف درهم، فقياسة أن تصرِّبه شيئًا
ودرهمين في القسم وهو نصف فيكون نصف شيء، ودرهما
يدخل شيئًا فالنصف نصف شيء بنصف شيء يعني درهما
يدخل نصف شيء فاضعه فيكون معك شيء يعدل درهماً

وهو أحد المالين والمال الآخر أربعة

فإن قال قسمت درهماً على رجال فاصيطم شيء ثم
زدت فيهم رجلاً ثم قسمت عليهم درهماً فاصيطم اثنين
من القسم الأول بسنس درهم فقياسة أن تصرِّبه عدد الرجال
الأولين وهم شيء في التخصيص الذي بينهم ثم تصرِّبه ما
الجمع في عدد الرجال الأولى و الأخرين ثم تقسم ما
الجمع على ما بين الرجال الأولى والأخرين فانه يخرج مالك
الذي قسمته فصار عدد الرجال الأولين وهو شيء في
السنس الذي بينهم فتكون سدس جذره ثم أضرب ذلك في
عدد الرجال الأولين والأخرين وهو شيء وواحد يكون سدس

٥٣
و تنضرب الأربعة الدراهم في خمسة وتسعة عشر جزءاً من خمسة وعشرين فيكون ثلثاً وعشرين درهماً وجزءاً من خمسة وعشرين وتضرب أربعة اجذار وتثليثاً في خمسة وتسعة عشر جزءاً من خمسة وعشرين فيكون اثني عشر جذراً وثاني عشر جزءاً من خمسة وعشرين وثلثين من جذر وثاني عشر جزءاً من خمسة وعشرين درهماً واربعة وتسعون وخمسين فتكون منها الدرهم الثلثة والعشرين والجزء من الخمسة والعشرين الذي كان مع المال فتبقى مائية وثمانية وثلثان وأربعون واربعون جزءاً من ستمائتين وخمسة والعشرين فتاخذ جذر ذلك وهو أحد عشر درهماً وثلث عشر جزءاً من خمسة وعشرين فتزيد على نصف الأجذار التي هي اثني عشر درهماً وأثني عشر جزءاً من خمسة وعشرين فيكون ذلك اربعة وعشرين وهو المال المتطلوب الذي تعزل ثلثه وربعه واربعة دراهم ثم تضرب ما بقي في مثله فيعود المال وزيادة اثني عشر درهماً.

فإن قال مال ضرته في ثلثه فبلغ خمسة فقيهه أن
خمسة أجزاء من اثني عشر من شيء الا أربعة دراهم
فتصير في مثلا فتكون الأجزاء الخمسة عشرة وعشرين
جزءا فتصير اثني عشر في مثلا نكون مائة واربعة واربعين
وذلك خمسة وعشرون من مائة واربعة واربعين من مال ثم
تضرب الأربعة الدراهم في الخمسة الأجزاء من
اثني عشر من شيء مرتين فيكون اربعين جزءا كل اثني
عشر منها شيء والأربعة الدراهم والأربعة الدراهم ستة عشر
درهما زائدة فتصير الأربعون الجزء ثلثة اجذار وثالث جذر
تاونس فيحصل معك خمسة وعشرون جزءا من مائة واربعة
واربعين جزءا من مال وستة عشر درهما الثالثة اجذار
وثلث جذر يعدل المال الولو وهو شيء وأثني عشر درهما
فاجبة وزد الثلاثة اجذار والثلاث على الشي واثني عشر
درهما فيصير أربعة اجذار وثالث جذر وأثني عشر درهما
فقابل به والف أثني عشر من ستة عشر يبقى اربعة دراهم
و خمسة وعشرون جزءا من مائة واربعين من مال يعدل
اربعة اجذار وثلثا فتحتاج ان تكمل مالك وعملائه اياه
ان تصرب جميع ما معك في خمسة وتسعة عشر جزءا
من اجزاء خمسة وعشرين فتصرب خمسة وعشرين في
خمسة وتسعة عشر جزءا من خمسة وعشرين فيكون مالا
جزء من جذر يعدل جذراً وثلث عشر درهم فاقت درهمين
من ثلثة عشر بدرهمين فيبقي أحد عشر درهما فاقت أحد
عشر جزء من جذر فيبقي نصف سدس جذراً واحد عشر
درهما يعدل نصف سدس مال فاكمله وذلك أن تضربه
في أثني عشر وتضرب كل ما معك في أثني عشر فيكون
مالم يعدل مائة وأثنيين وثلتين درهما وجزراً فقابل به
يضب أن شاء الله تعالى كما وصفت لك *
فإن قال درهم ونصف مقسوم على رجل وبعض رجل
نصاب الرجل مثل البعض فقياسه إن تقول الرجل
والبعض هو واحد وشيء فكانه قال درهم ونصف بين
واحد وشيء نصاب الواحد شيئين فأاصرب الشيئين في
الواحد والشيء فيكون مالين وشيئين يعدل درهما ونصفا
فردهما الي مال واحد وهو إن تأخذ من كل ما معك
نصفه فتقؤل مال وشيء يعدل ثلاثة أرباع درهما فقابل به
علي خوَاوَا وصفت لك في صدر الكتاب *
فإن قال مال عزلت ثلاثة وربعة أرباع درهما وضربت
ما بقي في مثله فعاد المال وزيادة أثني عشر درهما فقياسه
انك تأخذ شيئًا فتعزل ثلاثة وربعة فيبقي خمسة أجزاء من
أثني عشر جزءًا من شيء فتعزل منها أربع دراهم فتبقي
فيصير ممكح أربعة أتشاع مال وتسع تقراهم الأربعة أجاذر
يعدل جذرا فحز أربعة الأجاذر عليه أجاذر نيك في خمسة
أجاذر تعدل أربعة أتشاع مال وتسع تقراهم فاكمال مالكه وهو
إن تضرب أربعة الأتشاع في أشياء يربع فيكم مالا وضرب
تسعة تقراهم في أشياء يربع يكر عشرين وربعا ثم انضرب
الخمسة أجاذر في أشياء يربع فيكم واحد عشر شيئا وربعا
فيصير ممكح مال وعشرون درهم وربعا يعدل أدي عشر
جذرا وربعا فقابل بذلك كأنو ما وصفت لك في تصنيف
الاجذاران شاء الله *

فال قال مال تضرب ثلاثة في ربع فيعود المال قياسه
إن تضرب ثلاث شيء في ربع شيء فيكون نصف سدس
مال يعدل شيئا فالمال يعدل أثني عشر شيئا وهو جذر مائة
وأربعة واربعين *

فال قال مال تضرب ثلاثة ودربهما في ربع ودربهما
فيعد المال وزيادة ثلاثة عشر درهما قياسه إن تضرب ثلاث
شيء في ربع شيء فيكون نصف سدس مال وضرب
دربهما في ثلاث شيء فيكون ثلثي جذر ودربهما في ربع
شيء فيكون ربع جذر ودربهما في درهم درهما قياسه
نصف سدس مال ودربهما واحد عشر جزءا من أثني عشر

G
وكذلك لو قال مال تضرب جذره في أربعة إجذار، فيعود ثلثة أمثال المال وزيادة خمسين درهماً فقياسه أن تضرب جذراً في أربعة إجذار فيكون أموال عند ثلثة أموال وخمسين درهماً فاقت ثلثة أموال من الأربعة الأموال يبقى مال واحد يعدل خمسين درهماً وهو جذر خمسين مضرب في أربعة إجذار خمسين إليها فذاك مايتان يكون
ثلثة أمثال المال وزيادة خمسين درهماً فان قال مال تزيد عليه عشرين درهماً فيكون مثل
اثني عشر جذرة قياسه أن تكون مال وعشرون درهماً يعدل
اثني عشر جذراً فنصف الإجذار وأسبينها في مثلها تكون ستة
و ثلاثين فانقص منها العشرين الدراهم وخذ جذر ما بقي
فانتصه من نصف الإجذار وهو ستة فما بقي وهو جذر المال
وهو درهماً ومال أربعة
فان قال مال يعزل ثلثة وثلاثة دراهم ويضرب ما بقي
في مثله فيعود المال قياسه انك إذا القيت ثلثة وثلثة
داراه بقي ثلثاه الألفة دراهم وهو جذر نافسب ثلثي شيء
الألفة دراهم في مثله فتقول ثلاثان في ثلاثين أربعة أتماع مال
والاثنتين دراهم في ثلثي شيء جذران والألفة دراهم في ثلاثي
شيء جذران والألفة دراهم في الألفة دراهم تسعة دراهم
فان قال مال تصرفه في أربعة امتثال فيعود ثلث المال
الولقياسه انك إذا ضربته في اثني عشر مثله عاد المال وهو
نصف سدس من ثلث  
فان قال مال تصرفه في جذره فيعود ثلثة امتثال المال
الولقياسه انك إذا ضربت الجذر في ثلث المال عاد
المال فتكون هذا مال ثلاثة جذره وهو تسعة  
فان قال مال تصرف أربعة اجذاره في ثلاثة اجذاره
فيعد المال وزادته أربعة واربعين درهما فقياسي ان تصرف
اربعة اجذار في ثلثة اجذار فيكون اثني عشر مالا
واربعة واربعين درهما فالتة من الاثني عشر مالا
فبقي واحد عشر مالا عدل اربعة واربعين درهما فافتموها
عليها فتكون أربعة وهو المال  
فان قال مال تصرف أربعة اجذاره في خمسة اجذاره
فيعد مثلي المال وزادته ستة وثلاثين درهما فقياسي انك
تصرف أربعة اجذار في خمسة اجذار فيكون عشرين مالا
يعدل مالين وستة وثلاثين درهما فنلتقي من العشرين المال
مالين بمالين فيبقي ثمانية عشر مالا عدل ستة وثلاثين
درهما فنقسم ستة وثلاثين درهما على ثمانية عشر فيكون
القسم اثنيين وهو المال
كان قال، المال ثلثا خمسه مثل سبع جذر فان المال كله يعدل جذر ونصف سبع جذر فلجهذار أربعة عشر جزءًا من خمسة عشر من المال وقياسه أن تصرح ثلثي خمس المال في سبعه ونصف ليتم المال فانرب ما معك وهو سبع جذر في مثل ذلك فنصير المال يعدل جذر ونصف سبع جذر ونصير جذر واحد ونصف سبع فالم واحده وتسعة وعشرون جزءًا من مانة وستة وتسعين من درهم وثلثا خمسة يكون ثلاثين جزءًا من مانة وستة وتسعين وسبع جذر فان، أيضا ثلاثين جزءًا من مانة وستة وتسعين.

قال قال مال ثلثة ارباع خمسه مثل اربعة اخماس جذر فقيسه أن تزيد علي ثلثة ارباع خمسه مثل اربعة ليكون الجذر تاما ونذك ثلثة وثلثة ارباع من عشرين فاجعلها ارباعا كما يكون خمسة عشر من ثمانين فاقسم الثمانين على الخمسة عشر يكون خمسة وثالثا فذاك جذر المال الواحد.

فإن قال مال تصره في اربعة امتله فكون عشرين فقيسه.

اذن إذا ضربته في مثله كان خمسة وهو جذر خمسه.

فإن قال مال تصره في ثلاثة فكون عشرة فقيسه. اذن
في مثلها فتكون مائة ومالا الا عشرين شيئًا يعدل العشرة
الاجزاء فقابل بها على ما قد وصفت لك.
وكان ذلك أو قال عشرة قسمتها قسمين ثم ضربت احدهما
في الآخر ثم قسمت ما اجتمع من الفرض على فصل ما
بين القسمين قبل أن تنضرب احدهما في الآخر فخرج خمسة
وربما قياسة أن تأخذ شيئًا من العشرة فيبقي عشرة الا شيئًا
فاضرب احدهما في الآخر فيكون عشرة اجزاء المال الخمسة
فخرج من ضرب احدهماThan القسمين في الآخر ثم قسمت ذلك
علي فصل ما بين القسمين وهو عشرة الا شيئين فخرج من
المال المضرع وهو عشرة الا شيئين فهو ما
الشيئين خرج كل المال المضرع وهو عشرة الا شيئين.
فاضرب خمسة وربعا ومتى ضربت خمسة وربعا في عشرة
المالا الا الخمسة وربعا في عشرة الا شيئين يكаниثن
وخمسين درهما ونصفا الا عشرة اجذار ونصفا يعدل عشرة
اجذار الا مالا الا الخمسين والخمسين والنصف بالعشرة
الاجزاء ونصفا وربعا وثلثا والثلثا عشرة اجذار الا مالا الا
بالمال وزد المال على اثنين وخمسين درهما ونصف فيكون
معك عشرين جذرًا ونصف جذر يعدل اثنتين وخمسين
درهما ونصفا ومالا وقابل به على ما فسرنا في أول
الكتاب.
في تقسيم ستة أشياء ونصف يعد درهمين فاليات الواحد
اربعاً أجزاء من ثلاثة عشر من درهم وباقية ستة كل واحد
بجزءين من ثلاثة عشر من درهم فبلغ ذلك ثمانية وعشرين
جزءاً من ثلاثة عشر من درهم وذلك مثل فصل ما
بين الكيلين وهو تقفزان وفرقهما ستة وعشرون جزءاً وفصل
ما بين السبعين وهو جزءان فذالك ثمانية وعشرون
جزءاً
قال قال مالك ييذهما درهمان قسمت التليل علي الكثير
فانضب القسم نصف درهم فجعل أحد الماليين شيئاً والآخر
شيئاً ودرهمين فلما قسمت شيئاً علي شيء ودرهمين
خرج القسم نصف درهم وقد علمت أنك قديمت ما
خرج لك من القسم في المقاس عليه عاد مالك الذي
قسمته وهو شيء فقلل شيء ودرهمان في التصف الذي هو
القسم فيكون نصف شيء ودرهما يعد شيئاً فالتقيت نصف
شيء بنصف شيء وباقي درهما يعد نصف شيء فاضعه
يكون الشيء يعد درهمين والاربعاء
قال قال عشرين قسمتها قسمين فضربت احدهما في
عشرة والقسم الآخر في نفسه فاستوياً قياسه ان تضرب
شيئاً في عشرة فتكون عشرة أشياء ثم تضرب عشرة الأشياء
يعدل احداً وثمانين شيناً فاجبر الماية والملال بالعشرين الشيء، وزدها علي الوحد والثمانين فتكون ماية ومعلاً يعدل ماية جدر وجزءاً نصف الإجزار فتكون خمسين ونصفاً واضهماً في مثلها نصفين وخمسين، وخمسين وربع فنقض منها الماية فيبقي الفن وخمسماة وخمسين وربع فنقضها وهو تسعه وأربعون ونصف فنقضتها من نصف الإجزار وهو خمسون.

نصف فيبقي واحد وهو واحد القسمين.

قال قال عشرة أرقعة حنطة أو شعير بعثت كل واحد ما بين السبعين ومنهم ما بين الكيلين فخذ ما شئت فانه يجوز فانك اخذت أربعة وستة قلت بعث كل واحد من الأربعة بشيء فضحت أربعة في شيء فصار أربعة أشياء وبعث السنة كل واحد بمثل نصف الشيء الذي بعث به الأربعة وإن شئت بثلثه وإن شئت بربعه وما شئت فانه يجوز فاذ كأن بيعك الآخرين نصف شيء فاضرب نصف شيء في ستة فتكون ثلاثة أشياء فاجمعها مع الأربعة الأشياء فتكون سبعة أشياء تعديل ما بين الكيلين وهو فنيران وفصل ما بين السبعين وهو نصف شيء فتكون سبعة أشياء تعديل اثني ونصف شيء فتكون سبعة أشياء
الخمسة اللائية على عشرة إلا شيئًا وأخذت نصف ما خرج كان ذلك قسمك نصف الخمسة اللائية على العشرة إلا شيئًا فاختفت نصف الخمسة اللائية صار شيئين ونصفا وهو الذي تريد أن تقسمه على عشرة إلا شيئًا [يخرج] يعدل خمسين اللائية شيئًا لأنه قال نقسم إليه أحد القسمين مضروبًا في خمسة فتكون ذلك كله خمسين. وقد علمت أنك متي ضربت ما خرج لك من القسم في المقسم عليه عاد المال ومالك شيئان ونصف فاعرب عشرة إلا شيئًا في خمسين اللائية شيئًا فيه فتكون ذلك كله خمسة يعدل نصف شيء ونصفا فارد ذلك إليك مال واحد. فيكون ذلك مائة درهم ومال الإعشيرين شيئًا يعدل نصف شيء فاجبر ذلك المائة وزد العشيرين الشيء على نصف الشيء فنصب ذلك المائة ومال العشيرين الشيء ونصفًا نصف شيء ونصف شيء فنصب اللائي واتسعها في مثلها وانقس منها المائة وخذ جذر ما بقي وانقصه من نصف الأخذار وهو عشرة وربع فيبقي ثمانية وهو أحد القسمين

فال قال عشرة تسهمها قسمين فضربت أحد القسمين في نفسه فكان مثل الآخر أحد وثمانين مرة فقسم ذلك أن تقول عشرة إلا شيئًا في مثلها مائة ومال الإعشيرين شيئًا
الشيء فيكون معك صدقة وإربعة أموال وسدس مال يعدل
أحد وأربعين شيئا وثلثي شيء فاردد ذلك الي مال وقد
علمت أن المال الواحد من أربعة أموال وسدس هو خمسا
وخمس خمسا فخذ من جميع ما معك الخامس وخمس
الألف من أربعة وعشرون مال يعدل عشرة إجازة
لاه عشرة من أحد وأربعين شيا وثلثي شيء خمسا وخمس
خمسا نصف الإجازة وهي خمسة وأثريا في مثلها فتكون
خمسة وعشرين فانقسم منها الأربعة والعشرين التي مع
المال يبقى واحد فخذ جذره وهو واحد فانقسم منه نصف
الإجازة وهي خمسة فبقي أربعة وهو واحد التسنين * وأعلم
بان كل شينين تقسم هذا علي هذا وهذا علي هذا فانه إذا
ضيت الذي يخرج من هذا في الذي يخرج من هذا كان
واحداً ابداً *
فان قال عشرة قسمتها تسنين وضربت أحد التسنين
في خمسة وقسمته علي الامراث الثيت نصف ما اجتمع
معك وزدت علي المتمور في خمسة فكان خمسين درهما
فان قيس ذلك ان تأخذ شيا من العشرة فنصره في
خمسة فتكون خمسة اشياء مقسمة علي الباقين من العشرة
وهو عشرة لا شيئا مأخوذ نصفه ومعلوم انك إذا قسمت
ومال يعدل أحد عشر شيئًا نصف الأشياء فتكون خمسة ونصفًا فانصروا في مثلها فتكون ثلاثين وربعًا فانصروا منها الثامن والعشرين التي مع المال فيبقى إثنتان وربع فخذ جذر ذلك وهو واحد ونصف فأنصروا من نصف الإجذر يبقى أربعة وهو أحد التسعين

فإن قال عشرة قسمت تسعين قسمتهم ذلك علي هذا وهذا علي هذا فبقي ذلك دهرين وسدسًا * فقياس ذلك إذا ضربت كل قسم في نفسه ثم جمعتهما كان مثل أحد التسعين إذا ضربت أخاهما في الآخر ثم ضربت الذي اجتمع معك من الصرب في الذي بلغ القسم وهو أثنتان وسبع فنصب عشرة الأشياء في مثلها فتكون مائة ومال الأ عشرين شيئًا وأ traged شيئًا في شيء فيكون مالًا فجمع ذلك فيصير مائة ومائتين الأ عشرين شيئًا يعدل شيئًا مضروبًا في عشرة الأشياء وذلك عشرة إشية الأ مالًا مضروبًا في ما خرج من التسعين وهو أثنتان وسبع فتكون ذلك إحدا عشرتين شيئًا وثلاثي شيء الأ مالين وسديسا يعدل مائة ومالين الأ عشرين شيئًا فأضرب ذلك وزد مالين وسديسا على مائة ومائتين الأ عشرين شيئًا وزد العشرين شيء الناقصة من المائة والمليين على الواحد والعشرين الشيء وثلاثي
عشرين شيئا فيبقي ماية إلا العشرين شيئا يعدل اربعين درهما
فاجر الماية بالعشرين الشيء فإنها على الاربعين فيكون ماية
تعدل عشرين شيئا وأربعين درهما فالتقي العشرين من الماية
فيبقي ستون درهما تعدل عشرين شيئا فالشيء الواحد يعدل
ثلثة وهو أحد القسمين

وان قال عشرة قسمتة قسمين فنصبته كل قسم في نفسه
وجمعهما وزدت عليهما فصل ما بين القسمين من قبل ان
نشرهما فبلغ ذلك اربعة وخمسين درهما فإن قياسه ان
نسر عشرة الشيء في مثلها فتكون ماية ومال الا عشرين
شيئا وتصرف الشيء الثاني من العشرة في مثله فيكون مالا ثم
تجمع ذلك فيكون ماية ومالي الا عشرين شيئا وقال زدته
عليهما فصل ما بينهما قبل ان تسربهما فقلت فصل ما
بينهما عشرة الا شيئا فجمع ذلك ماية وعشرة ومالي الا
اثنين عشرين شيئا يعدل اربعة وخمسين درهما فادا جبرت
وقابلت قلته ماية وعشرة دراهم ومالي يعدل اربعة وخمسين
درهما وأثنين وعشرة شيئا فارد المالي الي مال واحد وهو
ان تأخذ نصف ما معك فيكون خمسة وخمسين درهما
ومال يعدل سبعة وعشرين درهما واحد عشر شيئا فالشيء
وعشرين من خمسة وخمسين فيه وعشرون درهما
باب المسائل المختلفة *

فإن مسألة فقال عشرة تسميتها تسميت ثم ضربت
أحدهما في الآخر فكان واحدا وعشرين درهما فقد علمت
أن أحد التسنين عن العشرة شيء وأخرى عشرة إذا شئت
فاضرب شيئا في عشرة إذا شئت فإثنان عشرة مالا
يدل واحدا وعشرين فاجبر العشرة الأشياء بالمال وزنه على
الواحد والعشرين فتكون عشرة أشياء تعدل واحدا وعشرين
درهما ومالا فالت ان نصف الاجذار فثقي خمسة فائضها في مثلا
تكون خمسة وعشرين فالت منهما الواحد والعشرين التي مع
المال فثقي اربعة فتأخذ جذرها وهو اثنان فائضهم من نصف
الجذار وهي خمسة يبقي ثلثا وذلك أحد التسنين.
وان شئت زدت جذر الأربعة على نصف الاجذار فتكون سبعة
وهو أحد التسنين وهذه المسألة التي تعمل بالزيادة
والنقصان *

وإن قال عشرة تسميتها تسميت فضربت كل قسم في نفسه
ثم الفيت الأدنى من الأكثر فثقي اربعون قياسه ان تضرب
عشرة إذا شئت في مثلا فتكون مالا ومالا الأعشرين شيئا
وتضرب شيئا في شيء فإثنان مالا فتائضه من المال والمال الا
مثلًا فتكون خمسة وعشرين فالتثناء الواحد والعشرين التي مع المال فيبقي اربعة فخذ جذرهما وهو اثنان فانقصه من نصف الإجذار التي هي خمسة فيبقي ثلثة وهو واحد التسنين والأخر سبعة فقد اخرجتك هذه المسألة الي أحد الأبواب الستة وهو

* أموال وعدد تعديل جذورا

المسألة السادسة * مال ضربت ثلثه في ربعه فعاد المال زيادة اربعة وعشرين درهما * فقيهنا ان تجعل مالك شيئا ثم تضرب ثلث شيء فربع شيء فتكون نصف سدس مال يعدل شيئا واربعة وعشرين درهما ثم تضرب نصف سدس مال في اثني عشر حتي تكون مالك فاعرب الشيء في اثني عشر يك اثني عشر شيء وضرب الاربعة والعشرين في اثني عشر فصير مركب مائتان وثمانية وثمانون درهما وأثني عشر جذرا يعدل مالا فنصف الإجذار تكون سته وأربوا في مثلها وزدها على مائتين وثمانية وثمانين فتكون ثلثمئة وأربعة وعشرين فخذ جذرهما وهو ثمانية عشر فزيد على نصف الإجذار وهي ستة فتكون ذلك اربعة وعشرين وهو المال فند اخرجتك هذه المسألة الي أحد الأبواب الستة وهو جذور

* عدد تعدل أموالا
الإجذر وانجزها في مثلها تكين اثنين عشر وربعا فنزاها على
الإحداد وهي مائتان وثمانية وعشرون فتكون مائتين واربعين
وربعا فنجذبه خمسة عشر ونصفا فانصه منه نصف الإجذر
وهو ثلثة ونصف فبقي اثني عشر وهو المال فقد اخراجت
هذى المسألة الي اجتمعت السبعة وهو الموار وحذور
تعدل عددا

والمسألة الخامسة * عشرة قسمتها قسمين ثم ضربت
كل قسم في نفسه وجمعهما فكانا ثمانية وخمسين درهما
قياسه ان تجعل احد القسمين شيئًا والآخر عشرة الا شئًا
فاضب عشرة الا شئًا في مثلها ف يكون مائة ومال الا عشرين
شيئا ثم تضرب شيئًا في شيء يكون مالا ثم تجمعهما فيكون
ذلك مائة ومالين الا عشرين شيئًا يعد ثمانية وخمسين
درهما فاجبر الالي ومالين بالعشرين الشيء الناقصة وزدها علي
الثمانية والخمسين يكين مائة ومالين يعد ثمانية وخمسين
درهما وعشرين شيئًا فاردد ذلك الي مال واحد وهو ان
تاخذ نصف ما معك فيكون خمسين درهما ومالا يعد
تسعة عشرة درهما وعشرة اشياء فقابل به وذلك اكتم
تلقى من الخمسين تسعه وعشرين فيبقي احد وعشرون ومال
يعدل عشرة اشياء نصف الإجذر تكون خمسة وانجربي في
التسمين شيئًا والآخر عشرة الأشياء ثم تقسم عشرة الأشياء علي شيء ليكون اربعة وقد علمت أنك متي ما ضربت ما خرج
لك من القسم. في المتسوم عليه عاد المال الذي قسمته والقسم في هذه المسألة اربعة والمتسوم عليه شيء فاضرب اربعة
في شيء فإن يكون اربعة اشياء تعدل المال الذي قسمته وهو عشرة الأشياء فاجبر العشرة بالشيء وزده علي الاربعة الأشياء
فيكون خمسة اشياء تعدل عشرة فاليه الواحد اثنان وهو
احده التسمين فقد اخراجت هذه المسألة الي أحد الإجابات
السنت وهو جذور تعدل عددًا

والمسألة الرابعة * مال ضربت ثلاث ودرهما في ربع
ودرهم فكان عشرين * قياسه ان تضرب ثلاث شيء في
ربع شيء فإن يكون نصف سدس المال وتضرب درهما في ثلاث
درهما يكون ثلاث شيء ودرهما في ربع شيء بربع شيء
ودرهما في درهما بدرهما فذلك كله نصف سدس المال وثلاث
شيء وربع شيء ودرهما يعدل عشرين درهما فالتسع
العشرين درهما بدرهما نتبقي سدس عشرة درهما تعدل نصف
سدس المال وثلاث شيء وربع شيء وكم مالك واكماله
ان تضرب كل ما معك في اثني عشر فيصير معك مال
وسعة اجذار يعدل مائتين وثمانية وعشرين درهما نصف
نفسه والباقي من العشرة أثنا عشر وهو القسم الآخر فقد أخرجته هذه المسألة الي أحد الأبواب ستة وهو أموال تعديل جذوراً فاعل ذلك

والمسألة الثانية عشرة قسمنا قسمت كل قسم في نفسه ثم ضربت العشرة في نفسها فكان ما اجتمع من ضرب العشرة في نفسها مثل أحد التسمين مصروب في نفسه مرتين وسبعة اتساع مرة أو مثل الآخر مصروب في نفسه ست مرات وربع مرة فقياس ذلك أن تجعل أحد التسمين شيئًا والآخر عشرة الشئين فتنضرب الشيخ في نفسه فيكون مالًا ثم في الاثنين وسبعة اتساع فيكون مالين وسبعة اتساع مال ثم تنضرب العشرة في مثليا فيكون ماية تعديل مالين وسبعة اتساع مال فارده الي مال واحد وهو تسعًا اجزاء من خمسة وعشرين جزء وهو خمس ورابعة اخماس الخمس فخذ خمس المائة واربعتهم اخماس خمسيا وهو ستة وثلاثين تعديل مالًا فخذ جذرها ستة وهو أحد التسمين والآخر اربعة لا سجالة فقد أخرجته هذه المسألة الي أحد الأبواب ستة وهو أموال تعديل عدداً

والمسألة الثالثة عشرة قسمنا قسمت ثم قسمت احدهما على الآخر فخرج القسم اربعة فقياسه أن تجعل أحد
باب المسائل الستي وقد قدمنا تقبل إبراز الحساب ووجوبه ست مسائل جعلتها أمثلة للسنة الإبراز المتقدمة في صدر كتابي هذا الذي أخبرت أن منها ثلثة لا تنص فيها الإجذار وذكرت أن حساب الجبر والمقابلة لا بد أن يخرجك إلى باب منها ثم اتبعنا ذلك من المسائل بما يقرب من الفهم وتجل في الموضوع وتسهل فيه الدلالان أن شاء الله تعالى

فالآلي من الستي نحو تولك عشرة قسمتاه تسمين فضربت أحد التسمين في الآخر ثم ضربت تجدهما في نفسه فصار المضرب في نفسه مثل أحد التسمين في الآخر أربع مرات* فقياسه ان تجعل أحد التسمين شيئًا والآخر عشرة الشيئًا فضربت شيئًا في عشرة الشيئًا فيكون عشرة أشياء الأملاء ثم تضربه في أربعة تنولك أربع مرات فيكون أربعة أمثال المضرب من أحد التسمين والآخر فيكون ذلك أربعين شيئًا الأربعة أموال ثم تضربه في شيء وهو أحد التسمين في نفسه فيكون ذالك يعدل أربعين شيئًا الأربعة أموال فاجبره بالربعة الأموال وزدها على المال فيكون أربعين شيئًا يعدل خمسة أموال فالمال الواحد يعد ثمانية اجذار وهو أربعة وستون جذرها ثمانية وهو أحد التسمين المضرب في
مايتن هو جذر ثماني مِاية وذلك ما اردنا أن نبين وهذه صورته.

وأما ميّة ومال الآشرين جذرا مجموعه خمسون وعشرة أجذار الآشرين فلم تستقيم له صورة لأنه من ثلثة اجناس مختلفة اموال وجذور وعدد وليس معها ما يعادلها فنصور وقد تمكننا لها صورة لا تحسن فاما انطلاقا باللفظ فبيّن وذلك انك قد علمت ان معك ميّة ومال الآشرين جذرا فلما زدت عليها خمسين وعشرة أجذار صارت ميّة وخمسين ومال الآشرين أجذار فهذه العشرة أجذار المزيدة جبرت من العشرين أجذار الناقصة عشرة أجذار فقته ميّة وخمسين ومال الآشرين أجذار وقد كان مع الميّة مال فلما نقصت مال الميّة والمالان المستثنين من الخمسين ذهب مال بمال وبقي عليك مال فصارت ميّة وخمسين الآشرين ولا عشرة أجذار وذلك ما اردنا أن نبين.
وأما علاقة جذر مائتين العشرين من عشرين الأجدر
مائتين فإن صورة ذاك خط ا ب وهو جذر مائتين وسأالي
نقطة ج هي العشرين المعلومة وتخرج من نقطة ب خط
الى نقطة د وتجعل العشرين وتحمل من ب الى نقطة د
مثل خط جذر مائتين وهو مثل خط ا ب وقد تبين لنا أن خط
ج ب هو ما بقي من العشرين بعد القاء جذر المايتين فاردنا
أن نقص خط حب من خط د فاخرجنا من نقطة ب
خطا الى نقطة ح وهو مثل خط ا ج الذي هو العشرين نصار
جميع خط ن ا مثل خط زب وخط ب د وقد تبين لنا أن
ذلك كله ثلثون وقطعنا من خط د مثل خط جب وهو
خط ج فتبين لنا أن خط ج هو ما بقي من خط رد الذي
هو ثلثون وتبين لنا أن خط ن جذر المايتين وخط رب
و ب جذر المايتين أيضاً فلما صار خط ج مثل خط جب
تبين لنا أن الذي نقص من خط زد الذي هو ثلثون جذرا
فانما علة جذر مائتين لا عشرة مجموعا الي عشرين لا جذر مائتين فإن صورة ذلك خط اب وهو جذر مائتين فمس ا الي نقطة ج هو العشرة الباقى من جذر مائتين هو الباقى من خط اب وهو خط جب ثم تخرج من نقطة ب خط الى نقطة د وهو خط العشرين وهو مثلا خط اج الذي هو عشرة فمس نقطة ب الي نقطة a مثل خط اب وهو جذر مائتين أيضا الباقى من العشرين هو من نقطة c الي نقطة d فلما اردنا ان نجمع ما بقي من جذر المائتين بعد طرح العشرة وهو خط جب الى خط د الذي هو عشرون الا جذر مائتين فقطعنا من خط b مثل خط جب وهو خط زة وقد كان تبين لنا ان خط اب الذي هو جذر مائتين مثل خط b فإن خط اج الذي هو العشرة مثل خط بز والباقي من خط اب الذي هو جب مثل الباقى من خط ب الذى هو زدنا علي خط ٩ خط زة فتبين لنا انه قد نقص من خط بد الذي هو عشرون مثل خط اج الذي هو عشرة وهو خط بز وباقي لنا خط زد وهو عشرة وذلك ما اردنا ان نبين 

وهذه صورة 

*
أرايتك في عمل الانعاف فما بلغ فاتسه علي اربعة وعلي ما اردت ان تقسم عليه وأعمل به كما عملت * وذكذلك اردت ثلاثة اجذار تسعة أو أكثر أو نصف جذر تسعة أو اقل أو ما كان فعلى هذا القياس فاعمله تنصب ان شاء الله تعالى *
وقد اردت ان تضرب جذر تسعة في جذر اربعة فاضرب تسعة في اربعة فيكون ستة وثلاثين فخذ جذرها وهو ستة وهو جذر تسعة مضروب في جذر اربعة * وكذلك لو اردت ان تضرب جذر خمسة في جذر عشرة فاضرب خمسة في عشرة فجذر ما بلغ هو الشيء الذي تريده * فان اردت ان تضرب جذر ثلاث في جذر نصف فاضرب ثلثا في نصف فتكون سدسا فجذر السدس هو جذر الثلاث مضروب في جذر النصف * وان اردت ان تضرب جذر سبعة تسعة في ثلاثة اجذار اربعة فاستخرج جذري تسعة علي ما وصنت لك حتي تعلم جذر اي مال هو وكذلك فاعل بثلاثة اجذار الرابعة حتي تعلم جذر اي مال هو ثم اصرف المالين احدهما في الآخر فجذر ما اجتمع لك هو جذري تسعة في ثلاثة اجذار اربعة وكذلك كلما زاد من الاجذار أو نقص فيلي هذا المثال فاعمل به *
فيكون جذر ما اجتمع مثل نصف جذر ذلك المال وكذلک ثلثة وأربعة أو مثل من ذلك أو أكثر بالغا ما بلغ في النقصان والاتعاف. ومثال ذلك إذا اردت أن تضع في جذر تسع أربعة أربعة في أثنين ثم في تسع ستة وثمانين فخذ جذر تسعثلث مرات تسعثلث وذلک لو اردت أن تضع جذر تسع ثلث مرات ينقرس فعلن ستة ثلث ثم في تسع ستة ينقرس ثلث وثمانين فخذ جذر تسعثلث في ثلث ثم في تسع ستة ينقرس واحد وثمانين فخذ جذر ستة وذلک جذر تسع مضاعفا ثلاث مرات فان اردت ان تأخذ نصف جذر تسع فاتخذ نصفا في نصف فاتخذ من نصف فاتخذ ثم تضرب ربعا في تسع ستة ينقرس أثنين وربعا فتحا جذرها وهو واحد ونصف وهو نصف جذر تسع وذلک ما زاد أو نقص من المال والاسم فهذا طريقة القسم. فان اردت ان تتقسم جذر تسع علي جذر اربعة فاتخذ تقسم للمثلثة علي اربعة فيكون أثنين وربعا فتجذرها وهم ما يصيب الواحد وهو واحد ونصف. فان اردت ان تقسم جذر اربعة علي جذر تسع فاتخذ تقسم المثلثة علي تسع فتكون اربعة تساع واحد فتجذرها ما يصيب الواحد وهو ثلثا واحد. فان اردت ان تتقسم جذر تساع علي جذر اربعة أو غيرها من المال فافتعف جذر التسعة علي ما.
زايداً أو ناقصاً مثل الأشياء في زيادة شيء فالضرب الأخير

ناظم ابداً * فاعلم ذلك وبالله التوفيق *

باب الجمع والنقصان * أعلم إن جذر مايتيين عشرة

جموع إلي عشرتين الإجذر مايتيين فإنه عشرساً * وجدز

مايتيين العشرة منقوض من عشرتين الإجذر مايتيين فهو ثلاثون

الجذري مايتيين وجدزا مايتيين هو جذر مثاني ماية *

ومائة ومال الأعشرين جذراً جموعاً إليه خمسون وعشرة اجذار

مالين فهو ماية وخمسون الأما واثنين عشرة اجذار *

ومائة ومال الأعشرين جذراً منقوض منه خمسون وعشرة اجذار

مالين فهو خمسون درهماً وثالثة أموال الأثنتين جذراً * وإن

مبين لك علة ذلك في صورة تودي إلي الباب إن شاء الله

تعالي * واعلم إن كل جذر مال معلوم أو اسم تريد أن

تضعبه ومعني اشعاثك إياك إن تصرسه في أثنين فينغي

تضرب أثناً في أثناً ثم في المال فيصير جذر ما اجتمع

مثلي [جذر] ذلك المال * وإن ارتدت ثلثة امثاله فاضرب

ثلثة في ثلثة ثم في المال فيكون جذر ما اجتمع ثلثة امثال

جذر ذلك المال الأول وكذلك ما زاد من الأبعاف أو نقص

فعلي هذا المثال نفسه * وإن ارتدت ان تأخذ نصف جذر

مال فينغي ان تضرب نصفاً في نصف فيكون ربعاً ثم في المال
فيكون عشرة اشياء المالا * ويقال عشرة شيء في شيء.

العشرة قلت شيء في عشرة اشياء زائدة شيء في شيء مال مزائد و الاعشرة في عشرة مئايت درهم ناقصة و الا عشرة في شيء بعشرة اشياء ناقصة فتقول مال الامة درهم

بعد ان قاللله به وذلك ان تضرح عشرة اشياء زائدة بعشرة اشياء ناقصة فيبقى مال الامة درهم *

ويقال عشرة درهم ونصف شيء في نصف درهم الا خمسة اشياء قلت نصف درهم في عشرة خمسة درهم زائدة ونصف درهم في نصف شيء بريع شيء زائد والا خمسة اشياء في عشرة درهم

خمسة جذورا ناقصة فيكون جميع ذلك خمسة دراهم الا تسعة واربعين جذورا وثلثة ارباع جذر ثم تضرب خمسة اجذر ناقصة في نصف جذر زاد فيكون مالين ونصفا ناقصة فذلك خمسة دراهم الا مالين ونصفا والتسعة واربعين جذرا

وثلثة ارباع جذر * قال قال عشرة شيء في شيء الا عشرة فكانه قال شيء وعشرة في شيء الا عشرة قلت شيء في شيء مال مزائد وعشرة في شيء عشرة اشياء زائدة والا عشرة في شيء عشرة اشياء ناقصة فذهبت الزيادة بالنقصان

وبقي المال والا عشرة في عشرة مئايت متصلة من المال فجميع ذلك مال الامة درهم * وكان ما كان من الضررب
قلت عشرة في عشرة ماية وعشرة في شيء عشرة اشياء وعشرة
في شيء عشرة اشياء اينما وشيء في شيء مال زايد فيكون مالية درهم وعشرين شيئًا ومال زايدا * وان قال عشرة الا شيئًا في عشرة الا شيئًا قالت عشرة في عشرة ماية والشيء في عشرة عشرة اشياء ناقصة والشيء في عشرة ماية مال الا عشرين شيئًا * وكذاك لو أنه قال لكت درهم الا سدس في درهم الا سدس يكون خمسة اسداس في مثلا وهو خمسة وعشرون جزءًا من ستة وثلثين من درهم وهو ثلاث
وسدس السدس وقياسه ان تضرب درهما في درهم فيكون درهما الا سدس في درهم بسدس ناقص والا سدس في درهم بسدس ناقص فيتبقي ثلثان والا سدس في الا سدس بسدس السدس زايدا وذلك ثلاث وسدس السدس * فان قال عشرة الا شيئًا في عشرة وشيء قالت عشرة في عشرة ماية والشيء في عشرة عشرة اشياء ناقصة وشيء في عشرة عشرة اشياء زايدة والا شيئًا في شيء مال ناقص فيكون ذلك ماية درهم الا مالا * وان قال عشرة الا شيئًا في شيء قلت عشرة في شيء عشرة اشياء والا شيئًا في شيء مال ناقص.
فالضرب الرابع ناقص * وهو مثل عشرة وواحد في عشرة
والاثنين فالعشرة في العشرة مائة والواحد في العشرة عشرة زايدة
والاثنان في العشرة عشرون زايدة والواحد في الاثنين آثلاثان زايدان
فذلك كله مائة واثنان وثلاثون * وإذا كانت عشرة الا واحداً
في عشرة الا واحداً فالعشرة في العشرة مائة والواحد الناقص في
العشرة عشرة ناقصة والواحد الناقص أيضاً في العشرة عشرة ناقصة
وذلก عشرون والواحد الناقص في الواحد الناقص واحد زايد
فذلک احده وثمانونون * وإذا كانت عشرة اثنان في عشرة
الواحد فالعشرة في العشرة مائة والواحد الناقص في
العشرة عشرة ناقصة والاثنان الزايدان في العشرة عشرون زايدون
ذلک كله مائة وثمانية * وإنما بنيت هذا ليستدل به على
ضرب الأشياء بعضها في بعض إذا كان معها عدد أو استثنىت
من عدد أو استثنى منها عدد * فإذا قيل لک عشرة اللؤلؤ
وعني الشيء الجذر في عشرة فاضرب عشرة في عشرة يكون
مائة واللؤلؤ في عشرة يكون عشرة اجذار ناقصة فتصل مائة الا
عشرة أشياء * فإن قال عشرة وشيء في عشرة فاضرب
عشرة في عشرة يكون مائة وشيئاً في عشرة بعشرة أشياء زايدة
يكون مائة وعشرة أشياء * وإن قال عشرة وشيء في مثلها
ووجدنا كل ما يعمل به من حساب الجبر والمثبطة لا بد أن
يجترجج أن أحد الباب ستة التي وصفت في كتابي هذا
* وقد اتبعت على تفسيرها فاعرف ذلك
باب الضرب * وهى تعرف كيف تضرب الأشياء وهي
المجذور بعضها في بعض إذا كانت منفردة او كان معها عدد او كان
مستثنا منا عدد او كانت مستثنة من عدد وكيف تجمع
بعضها الى بعض وكيف تنقص بعضها من بعض * ان علم انه لا
تقد لكل عدد يضرب في عدد من ان تضافه احد العددتين بعدد
ما في الآخر من الاحاد * فذاك كانت عقود ومعها احاد او
مثبطة منها احاد فلا بد من ضربها أربع مرات العقود في
العقود والعقود في الاحاد والحاد في العقود والحاد في الاحاد
* فذاك كانت الاحاد التي مع العقود زائدة جميعاً فالضرب
الرابع زائد ايضاً * وإذا كان احدهما زايداً والاخر ناقصاً
الإجذار الذي هو واحد ونصف في مثله وهو اثنان وربع ثم زدنا في خط ط مثل خط آه وهو خط طل فصار خط ح ل مثل خط آه وخط كن مثل خط طل وحدث سطح مربع متساوي الأضلاع والزوايا وهو سطح ح م وقد تبين لنا أن خط آه مثل خط م ل وخط آه مثل خط ح ل فبقي خط ح ج مثل خط ن ر وخط م ن مثل خط طل فنفصل من سطح دب مثل سطح كن وقد علمنا أن سطح أر هو الأربعة الزائدة على الثلثة الإجذار فصار سطح أن وسطح كن مثل سطح أر الذي هو الأربعة العدد فتبين لنا أن سطح ح م هو نصف الإجذار الذي هو واحد ونصف في مثله وهو اثنان وربع وزيادة الأربعة التي هي سطح أن وسطح كن وقد بقي لنا من ضلع المربيعة الأولة التي هي سطح أر وهو المال كله نصف الإجذار وهو واحد ونصف وهو خط ح ج فإذا زدناه على خط آه الذي هو جذر سطح ح م وهو اثنان ونصف [وزدنا عليه خط ح ج الذي هو نصف الثلثة الإجذار وهو واحد ونصف] فبلغ ذلك كله اربعة وهو خط آه وهو جذر المال الذي هو سطح آه وهذه صورته وذلك ما اردننا ان نبين. *
الذي هو نصف الإجذاري. خط أب وهو ثلثة وهو جذر المال.

الآن إذا زدته على خط جد الذي هو نصف الإجذار، بلغ ذلك سبعة وهو خط رج ويتكون جذر المال أكثر من هذا المال إذا زدت عليه واحداً وعشرين عشرين مثل عشرة إجذار. وهذا صورته وذلك ما ارتدنا ان نبين

وأما ثلثة الإجذار وأربعة من العدد يعدل مالاً فاناً نجعل المال أربعاً سبعاً، يجعل الإجذاري متباين الإجذار والزوايا، وهو سطح أب هذا السطح كله يجمع الثلثة الإجذار وأربعة التي نذكرها وكل سطح أربعاً فان أحد أضعافه في واحد جذرناه فقطعنا من سطح أب سطح د فجعلنا أحد أضعافه الذي هو ج الثلثة التي هي عدد الإجذار وهي مثل رد فتين لنا أن سطح دب هو الأربعة المزدوجة على الإجذار فقطعنا ضلع دج الذي هو ثلثة إجذار بنصفين على نقطة ج ثم جعلنا منه سطحاً سبعاً وهو سطح د وهو ما كان من ضرب نصف
فحدين لنا أن خط ج تظهر مثل خط ح وقد تبين لنا أن الخط ح مثل خط ج قد نرى علي خط ح على استمالة مثل فصل ج ح علي خط ليبنبع السطح فصار خط ط كل مثل خط كتم وحدث سطح مربع متساوي الإقلاع والزوايا وهو سطح م ط وقد كان تبين لنا أن خط ك تخمة وضحه وهنا ما اجتمع من ضرب نصف الإجذار في مثلها وهو تخمة في خمسة يكون خمسة وعشرين * وقد كان تبين لنا أن سطح دب هو الواحد والعشرون التي زبدت علي المال فقطاعنا من سطح تابا * واخذنا من خط كتم خط ككل وهو مثل خط ح فتبين لنا أن خط ط كل مثل خط م وفصل من خط كتخمه لكن وهو مثل خط ح فصار سطح م ب مثل سطح نابا فتبين لنا أن سطح دب م زيدا عليه سطح م ب مثل سطح دب وهو واحد وعشرون وقد كان سطح م تخمة وعشرين فلما نقصنا من سطح م دب وسطح م الذي هما واحد وعشرين بقي لنا سطح صغير وهو سطح رتم وهو فصل ما بين خمسة وعشرين واحد وعشرين وهو أربعة وجذره خط رج وهو مثل خط ح وهو اثنان * فإن نقصهما من خط ح ج
علي تسعة وثمانين ليتم السطح الأعظم الذي هو سطح رّم فبلغ ذلك كله اربعة وستين فاحذنا جذرهما وهو ثمانية وهو أحد اضلاع السطح الأعظم فادا نقصنا منه مثل ما زدنا عليه وهو خمسة بقي ثلث وهو ضلع سطح آب الذي هو المال وهو جذر

وأما مال واحد وعشرون درهما يعدل عشرة اجذار فانا

جعل المال سطحا مربعا مح子ども الإضلاع وهو سطح آن ثم نصم اليه سطحا مستوازي الإضلاع عرضة مثل احد اضلاع سطح آن وهو ضلع د وسطح دب فصار طول السطحين جميعا ضلع جره

وقد علمنا ان طوله عشرة من العدد لن كل سطح مربع متساوي الإضلاع والزوايا فان احد اضلاعه مضروب في واحد جذر ذلك السطح وفي اثنين جذراه فلما قال مال واحد وعشرون يعدل عشرة اجذار علمنا ان طول ضلع رّج عشرة اعداد لن

ضلع جد جذر المال نقسمنا ضلع جره بنصفين على نقطة
ليتم لنا بناء السطح الأعظم بما نقص من زواياه الأربع إلى كل عدد يضرب ربعه في مثلث ثم في أربعة يكون مثل ضرب نصفه في مثلث فاستغننا بضرب نصف الإجذار في مثلث عن الربع في مثلث ثم في أربعة وهذا صورته

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

وله إيننا صورة أخرى تودي الي هذا وهي سطح أب وهو المال فارتنا إن نزيد عليه مثل عشرة أجذار فنصفنا العشرة فصارت خمسة فصيناها سطحين علي جنبي سطح أب وهما سطحا جد فنصار طويل كل سطح منها خمسة أذرع وهو نصف العشرة الإجذار وطرحه مثل ضلع سطح أب فبقيت لنا مربعة من زوايا سطح أب وهي خمسة في خمسة وهي نصف العشرة الإجذار التي زدناها علي جنبي السطح الول فعلما أن السطح الول هو المال وان السطحين الذين علي جنبتيهما عشرة أجذار فذكذك كله تسعة وثلاثون وباقي إلي تمام السطح الأعظم مربعة خمسة في خمسة فذكذك خمسة وعشرون فنذناها
فيه جذر وكل ضلع من اضلاعه إذا ضربته في عدد من الأعداد فما بلغت الأعداد فهي أعداد جذور * كل جذر مثل جذر ذلك السطح فلما قيل أن مع المال عشرة أجزاء اخذنا ربع العشرة وهو ثمنان ونصف وضربنا كل ربع منها مع ضلع من اضلاع السطح فصار مع السطح الأول الذي هو سطح اب وعبره ثمانية سطح متساوي الاضلاع مجيبا أيضا ناقص في زوايا الأربع في كل زاوية من النقاط ثمانيا ونصف في ثمانين ونصف فصار الذي يحتاج إليه من الزيادة حتى يتربع السطح ثمانية ونصف في مثله أربع مرات ونحاس ذلك جميعه خمسة وعشرون * وقد علمنا أن السطح الأول الذي هو سطح المال والربعة السطوح التي حوله وهي عشرة أجزاء هي تسعة وثمانين من الأعداد * فإذا زداها عليها الخمسة والعشرين التي هي المربعات الأربع التي هي على زوايا سطح اب تم تربيع السطح الأعظم وهو سطح ده وقد علمنا أن ذلك كله أربعة وستون واحد اضلاعه جذره وهو ثمانية فإذا نقصنا من الثمانية مثل ربع العشرة مرتين من طريق ضلع السطح الأعظم الذي هو سطح ده وهو خمسة بقي من ضلعه ثلاثة وهو جذر ذلك المال * وإننا نصفنا العشرة الأجذار وضربناها في مثلها وضربناها على العدد الذي هو تسعة وثمانون
مثل نصف الإجذار سوا لا زيادة ولا نقصان وكل ما ان นอกจาก من مالين أو أكثر أو اقل فاردته إلي مال واحد كتحوما بينت لك في الباب الأول *

واما الجذور والعدد التي تعدل الأموال فتحو تولك ثلثة 

الجذور وأربعة من العدد يعدل مالا فقيمةه ان تنصف الإجذار 

فتكون واحدة ونصفا فانضما في مثلها فتكون اثنين و ربعا نزدها 

على الربعة فتكون ستة و ربعا فلذ جذرهما وهو اثنان ونصف 

نفسه عليه نصف الإجذار وهو واحد ونصف نفسو اربعة وهو 

جذر المال الملال ستة عشر وكل ما كان أكثر من مال او اقل 

فاردةه الي مال واحد *

فهذه السنة الضروب التي ذكرتها في صدر كتابي هذا وقد 

اتبعت عليها تفسيرها واخبرت ان منها ثلثة ضروب لا تنصف 

فيها الإجذارود بنت تياسها واضطرارها * فاما ما يحتاج 

فيه الي تنصيف الجذور من الثلاثة الابواب الباقة فقد وسعته 

باباب صحيحة وصرت لكل باب منها صورة يستدل بها علي 

الصلة في التنصيف *

فاما علة مال وعشرة اجذار يعدل تسه وثلثين درهما 

فصورة ذلك سلط مربع محجب الاضلع وهو المال الذي تريد 

ان تعريه وتعريف جذره وهو سلط اب وكيل ضلع من اضلاعه
عشر ونصفه عينية * وكذاك فانفعل جميع ما جاءَت
من الأموال والجزور وما عادلها من العدد يصب أن شاء الله
وأما الأموال والعدد التي تعدل الجزور فانكو تولك مال
واحد وعشرون درهما من العدد يعدل عشرة إجذارة ومعاده
أي مال إذا زدت عليه واحدا وعشرين درهما كان ما اجتمع
مثل عشرة إجذار ذلك المال * فقياسه أن تتصف الإجذار
فيكون خمسة فاضريها في مثلها يكون خمسة وعشرين فانقص
منها الواحد والعشرين التي ذكر اثنتها مع المال فيبقى اربعة
فخذ جذرها وهو اثنان فانقصه من نصف الإجذار وهي خمسة
فيبقى ثلث وهو جذر المال الذي تريد ومال تسعه وان
شيت فنذ الجذر علي نصف الإجذار فيكون سبعة وهو جذر
المال الذي تريد ومال تسعه وأربعون * فإذا وردت
عليك مسألة تخرجك الي هذا الباب فانتهي صوابا
بالزيادة فإن لم تكن فيي بالنقصان لا سماحة وهذا الباب
يعمل بالزيادة والنقصان جميع وليس ذلك في غيره من
الابواب الثلاثة التي تحتاج فيها الي تتصف الإجذار
واعلم اناك إذا نصفت الإجذار في هذا الباب وضربتها في
مثلها فكان مبلغ ذلك اقل من الدراهم التي مع المال
فالمسألة مستحيلة وإن كان مثل الدراهم بعينها فيجذر المال
إذا جمعا وزيد عليهما مثل عشرة إجذار أحدثها بلغ ذلك
ثمانية وأربعين درهماً ف ينبغي أن تردها المالين الي مال واحد
وقد علمت ان مالاً من مالين نصفهما نفارد كل شيء في
المسألة إلى نصفه فإنها قال مال وخمسة إجذار يعدل أربعة
وعشرين درهماً ومنعه أي مال إذا زدت عليه خمسة إجذار
بلغ ذلك أربعة وعشرين فنصف الأجذار نكون أثين
ونصفاً فاضراً بها في مثلها ي تكون ستة ورباعاً فنزوها على الأربعة
والعشرين فتكون ثلثين درهما ورباعاً فخذ جذرها وهو خمسة
ونصف فانقص منها نصف الأجذار وهو أثان ونصف تقبي
ثلثة وهو جذر المال والمال تسعة * وكذا لقول نصف
مال وخمسة إجذار يعدل ثمانية وعشرين درهماً فمعنى
ذلك أي مال إذا زدت على نصفه مثل خمسة إجذار
بلغ ذلك ثمانية وعشرين درهما فتريد ان تكمل مالك
حتى يبلغ مالاً تاماً وهو ان تضيفه فاعقه واعفع كلما
معك مما يعادله فيكون مالاً وعشرة إجذار يعدل ستة
وخمسين درهما فنصف الأجذار تكون خمسة فاضراً بها في مثلها
تكون خمسة وعشرين فنزوها على السناو الخمسين تكون احداً
وثمانية فخذ جذرها وهو تسعة فانقص منها نصف الأجذار وهو
خمسة فتبقي أربعة وهو جذر المال الذي ارتدت المال ستة
اربعة اجذار تعدل عشرين والجزر الواحد يعدل خمسة والمال الذي يكون منه خمسة وعشرون * وكتوابك نصف جذر يعدل عشرة فانجزر يعدل عشرين والمال الذي يكون منه أربعاءا * 
وجدته هذه الصررحة الثلثة التي هي الجذور والاموال والعدد يقترب فيها ثلثة اجذار متقترنة وهي اموال وجذور تعدل عدداً واموال وعدد تعدل جذورا وجذور وعدد تعدل اموالا *
فاما الموال والجذور التي تعدل العدد فنثل تولك مال وعرشة اجذار يعدل تسعة وتئلين درهما ومعناه أي مال إذا زدت عليه مثل عشرة اجذار بلغ ذلك كله تسعة وتئلين * فقياسه ان تنصف الاجذار وهي في هذه المسالة خمسة فنحربها في مثلها نفنك خمسة وعشرين فتزدها على التسعة والثلاثين فيكون اربعة وستين فتاخذ جذرة وثامانية فتنقص منه نصف الاجذار وهو خمسة فيبقي ثلثة وهو جذر المال الذي تريد والممال تسعة * وكذلك لو ذكر مالين أو ثلثين أو اثنين أو أكثر فارده الي مال واحد وارد ما كان منه الاخاذار والعدد الي مثل ما ردت إليه المال * وهو نحو قولك مالان وعشرة اجذار يعدل ثامانية وأربعين درهما ومعناه أي مالين
فاما الأموال التي تعدل الجذور فمثل تولك مال يعدل خمسة اجذار فجذر المال خمسة والمال خمسة وعشرون وهو مثل خمسة اجذار * وكتولك ثلاث مال يعدل اربعة اجذار فالمالي كله يعدل اثني عشر جذرا وهو ماية اربعة واربعون وجذرة اثني عشر * وكتولك خمسة اموال تعدل عشرة اجذار فالمالي الواحد يعدل جذرين وجزر المال اثنتان والمال اربعة * وكذلك ما كثر من الأموال أو قل يرَد الى مال واحد وكذلك يفعل بما عادلها من الاجذار بناءً على مثل ما يرَد الى المال *

واما الأموال التي تعدل العدد فمثل تولك مال يعدل تسعة فهو المال وجزرة ثلثة * وكتولك خمسة اموال تعدل ثمانين فالمالي الواحد خمس الثمانين وهو ستة عشر * وكتولك نصف جذراً في المال يعدل ثمانية عشر فالمال يعدل ستة وثلاثين وجزرة ستة * وكذلك جميع الأموال بايدها وناتجها ترَد الى مال واحد وإن كانت أقل من مال زيد عليها حتي تكون مالاً تاماً وكتولك تفعل بما عادلها من الأعداد *

واما الجذور التي تعدل عدداً فكتولك جذار يعدل ثلاثة من العدد فالمالثلثة والمال الذي يكون منه تسعة * وكتولك
وإلي لما نظرت فيما يحتاج إليه الناس من الحساب وجدت جميع ذلك عددًا ووجدت جميع الأعداد انتُركبت من الواحد وال واحد داخل في جميع الأعداد. ووجدت جميع ما يلفظ به من الأعداد ما جاور الواحد أي العشرة يخرج صخر واحد ثم تثنى العشرة وتثنى كما فعل بالواحد فيكون منها العشرون والثلاةون أي تمام العヤة ثم تثنى العية وتثنى كما فعل بالواحد وبالعشرة أي الألف ثم كذلك يريد الألف عند كل عدد إلى غاية المدرك من العدد. ووجدت الأعداد التي يحتاج إليها في حساب الجذر والمعتابة على ثلاثة ضروب وهي جذور وأموال عدد مفرد لا ينسب إليها جذر ولا مال * فالجذر منها كل شيء مصروب في نفسه من الواحد وما فوقه من الأعداد وما دونه من الكسور * والمال كله اجتمع من الجذر المضروب في نفسه والعدد المفرد كل متعلق به من العدد بالنسبة إلى جذر ولا مال * فن هذى المصروب الثلاثة ما يعدل بعضهم بعضا وهو كقولك اموال تعدل جذورا * وأموال تعدل عددًا * وجذور تعدل عددًا
اما رجل سبق الي ما لم يكن مستخرجا قبله فورئه من
بعد، واما رجل شرح مما اباقا الاولون ما كان مستغلفا فارض
طريقه وسلمه مسلكه وقرب مأخذه واما رجل وجد في بعض
الكتب خلاا فلم شعه واقام اوده واحسن الظن بصاحبه غير
زائد عليه ولا مفتكح من ذلك بكف نفه *
وقد شجعني ما قفل الله به الإمام الماهون امير المومنين
مع الخلقية التي جاز له ارتها وأكره بلباسها وحلاز برزتها
من الرغبة في الادب وتقرب اهله وادناءهم ووسط كنفه لهم
ومعonte اياه علي اثواب ما كان مستبهما وتسيل ما كان
مستوعبا علي ان الفت من حساب الجبر والمقابلة كتابا
مختصرا حاررا للطيف الحساب وجليلا لاما يلزم الناس من
العاجزة اليه في موارثهم ووصاياه وفي مقاتمهم واحكامهم
وتجارتهم وفي جميع ما يتعاملون به بينهم من مساحة الأراضي
وكري الاهوار والهندسة وخير ذلك من وجهه وفنونه مقدما
لحسن النية فيه وراجيا لن يبذل اهل الادب بفضل ما
استودعوا من نعم الله تعالى وجليل الية وجميل بليه عندهم
منزلته وبالله توفيق في هذا وفي غيره عليه توكلت وهو راب
العرش العظيم وضلي الله علي جميع الانبياء والرسلين *
بسم الله الرحمن الرحيم

هذا كتاب وضعه محمد بن موسي الخوارزمي انتهى بان قال الله في نعمة بما هو أهله من محامدته التي باداء ما افترض منها علي من يعبد من خلقه تقع اسم الشكر وتنصّب العزلة في غير إقرارا بربوبية وتدلا لعذابه وخشوعا لعظمته بعث محمد صلى الله عليه وسلم آله وسلما بالنبيه علي حيو مئات من الرسول وتنكر في الحق ودرس من الهدي فتغبر به من العمى واستبند به من الملكة وكثير به بعد القلعة ولف به بعد الشناد تبارك الله ربينا وعلي جده وتقدست اسماؤه ولا الله غيره صلى الله علي محمد النبي وآله وسلم.

ولم تنزل العلماء في الأزمة الخالية والأمم العاضية يكتبون الكتب مما يصنفون من صنف العلم ووجهة الحكمة يظرا لمن بعدهم واحساسا للأجر بقدر الطاقة ورجاء أن يلتجهم من إجر ذلك وذكرة وذكورة وبيغيم لمن لسان الصدق ما يضفر في جنبة كثير مما كانوا يتكلمونه من الرؤونة وتحملونه على أنفسهم من المشقة في كشف اسرار العلم وفاته.
الكتاب المختصر
في حساب التجبر والمقابلة

تصنيف

الشیخ الأجل ابن عیسی عبد الله محمد بن موسی
الخوارزمی

طبع في مدينة لندن
سنة 1830 المنسية
الكتاب المختصر
في
حساب الجبر و المقابلة